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Major E. H. HILLS, C.M.G., D.Sc., F.R.S., PRESIDENT, in the Chair.

Stanley Herbert Buckland, Walsingham House, Whiteley Road, Upper Norwood, Surrey;
Rev. Charles Evans, Elm House, Brenchley, Kent;
William Ernest Hartley, M.A., Chief Assistant, Cambridge Observatory;
Capt. Ronald L. Jones, Box 230, British Post Office, Shanghai, China;
Capt. Henry George Lyons, R.E., F.R.S., 5 Heathview Gardens, Roehampton, S.W.;
Rev. Edward O'Connor, S.J., M.A., Stonyhurst College, Blackburn, Lancashire; and
Henry Spencer Toy, B.Sc., Ashville College, Harrogate, and Helston, Cornwall;

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

R. A. Fisher, 56 Kempshott Road, Streatham, S.W. (proposed by F. J. M. Stratton);
John Spedding Fletcher, London County Council School of Engineering and Navigation, High Street, Poplar, E. (proposed by E. W. Owens);

John Guild, A.R.C.S., D.I.C., Imperial College of Science and Technology, South Kensington, S.W. (proposed by A. Fowler); and

Charles P. Olivier, M.A., Ph.D., Agnes Scott College, Decatur, Georgia, U.S.A. (proposed by Cleveland Abbe).

Fifty-two presents were announced as having been received since the last Meeting, including, amongst others:—

A. Auwers: Bearbeitung der Bradley'schen Beobachtungen an den alten Meridianinstrumenten der Greenwicher Sternwarte, Band 2, presented by Prof. Auwers. Miss M. A. Blagg: Collated list of lunar formations named or lettered in the maps of Neison, Schmidt, and Mädler, presented by Prof. Turner. Oxford Astrographic Catalogue, vol. viii., part 1, presented by the University Observatory, Oxford. H. H. Turner: Tables for facilitating the use of Harmonic Analysis, presented by the author.

Photograph of the late Sir William Huggins, from the portrait by the Hon. John Collier, presented by Lady Huggins. Spectroheliogram taken at the Meudon Observatory, presented by M. Deslandres. Photograph of the Memorial erected on the site of Captain Cook's astronomical station in Tahiti, photographed and presented by B. G. Corney.

The Mean Latitudes of the Moon and Sun. By Ernest W. Brown, M.A., Sc.D., F.R.S.

1. It is well known that Hansen added $-1''$ to all his latitudes in order to account for a supposed difference between theory and observation which he ascribed to a difference between the centre of figure and centre of mass of the Moon. Newcomb, in his last paper, characterised this procedure as one of the most unfortunate steps that Hansen took. But at the end of his investigation Newcomb himself found a well-determined difference of $-0''{\cdot}36$ which would have to be added to the latitudes or declinations if these were to agree with their observed values. This quantity, whatever its cause, must therefore be considered as one of the arbitrary constants of the Moon's orbit to be determined by observation.

The tabular latitude is particularly simple to correct for the purposes of the comparison. It contains but few long-period terms, and these are easy to determine with the necessary accuracy; planetary action and unknown forces either give short-period terms

or are likely to do so. On the other hand, the observations are liable to be affected by two kinds of systematic errors arising from the irregular shape of the Moon's limb and the personal equation involved in making a setting. With occultations the former error may become systematic if the irregularities are so. Observations of Mösting A and the Harvard photographs remove the difficulty a step farther back in the fact that their reduction requires the determination of the centre of the Moon from observations of the limb, but the personal equation can be either neglected or is capable of being determined.

2. It is therefore necessary to find out what value the Greenwich observations will give, or, rather, what systematic correction must be applied to the measured declinations in order that the Moon's mean orbit may coincide with the ecliptic. Most of the useful material (1847–1901) has been gathered together by Dr. Cowell,* and it was not difficult to supply the remainder up to 1911 from the annual volumes of the Greenwich observations. The results are given in the second column of Table II. This has to be corrected for tabular and well-ascertained observational errors: the chief trouble lay in finding out what the latter should be. It did not seem as if the collection of the observations, from 1750 to 1846 could add anything to the accuracy of the results. The largest errors all arise from the observations, and the gain for the determination of a fixed constant from a long series of comparatively inaccurate observations was out of all proportion to the work of collecting and correcting them. The errors of the longitude were sufficient to show this. The mean residuals dealt with here are of the order $0''{\cdot}1$, while those of the longitude were of the order of $1''$, and the early observations of the latitude might easily be affected with errors of this magnitude.

3. The result obtained, an addition of $-0''{\cdot}51$ to the declinations, is somewhat larger than Newcomb's adopted value, but it agrees with that which he deduced from immersions alone. The necessity for limiting the time which should be spent on this constant has compelled me to leave aside all investigation of personal equation; but I believe that if this were undertaken, it would give a much more homogeneous set of residuals, though it is doubtful if the general mean would be much altered.

The comparisons with the Sun were, in the first instance, made for the purpose of testing the corrections. Some interesting features appeared, and they are discussed with some detail. There is no evidence of any periodic change in the differences between the residuals deduced from the two bodies, but systematic errors extending over many years are brought out.

The origin of the average difference of half a second is also discussed. The exhaustion of the possibilities leads us back to Hansen's original hypothesis, which does not differ fundamentally from that which seeks to account for the difference by systematic irregularities of the Moon's limb.

* *M.N.*, vol. lxv. p. 726.

Tabular Corrections.

4. The only sensible additions to Hansen's tables affecting the mean latitude found after a comparison of Cowell's transformation * of Hansen's tabular theory with my theory were:

$$\begin{aligned}
&-0''\cdot16 \sin(F - l)\\
&-0\cdot03 \sin(-F - l + 2D - 3T + 3V)\\
&-0\cdot05 \sin(-F - l + 2D + 2T - 2J)\\
&+0\cdot02 \sin(-F - l + 2D + 2T - 3J - 8^\circ)\\
&+0\cdot02 \sin(F - l + D)\\
&-0\cdot01 \sin(F - l - D)\\
&-0\cdot03 \sin(2D - F + T - J)
\end{aligned}$$

The last three are "allied" terms. The importance of these has been emphasised by Dr. Cowell in his papers on the observed motion of the Moon. Although of short period, they produce an apparent effect owing to the clustering of all observations round full Moon. He has shown † that the tabular minus observed correction due to an erroneous term in the tables, $a \sin(a \pm D)$, is $-\frac{1}{2} a \sin a$. The effect, due to this cause, of the three terms is so small that it has been neglected. The other four terms were tabulated by Mr. Hedrick, and their sum is given in the third column of Table II.

Observational Corrections.

5. *Corrections to the Declinations.*—The Greenwich declinations have been the subject of several papers. The results given by Newcomb ‡ are in available form for the years 1851–87. These contain the corrections for reduction to Pulkowa refractions, to the colatitude 38° 31′ 21″·90 and to the R − D discordance and flexure in the form $b \sin$ Z.D., where $b = +0''\cdot30$ for 1851–77, $+0''\cdot51$ for 1878–82, $+0''\cdot68$ for 1883–87. I have adopted these with only one change, namely, to the colatitude 21″·80, which demanded the further addition of $-0''\cdot10$ to the observed north polar distances.

For the years 1847–50, when the mural circle was used, the only correction has been $-0''\cdot24$ for reduction to the Pulkowa refractions and the colatitude 21″·80. In any case these four years have very little influence on the final value of the mean latitude, and they have been given less weight on account of the doubt about the proper instrumental corrections to be applied.

For the years 1888–1905 I have corrected the observations by the addition of $-0''\cdot34$ for reduction to the Pulkowa refractions and to the colatitude 21″·80. The corrections to the coefficient of sin Z.D. for the R − D discordance have been adopted from the Greenwich ten-year catalogue of 1890 (1888–96) and that of 1900 (1897–1905), the adopted values being $+0''\cdot58$ and $0''\cdot60$, respectively. The observations for 1907–11 are left unchanged,

* *M.N.*, vol. lxiv. p. 165. † *M.N.*, vol. lxiv. p. 413.
‡ *Amer. Ephem. Papers*, vol. ii. pp. 450–2.

since the Pulkowa refractions, the R – D coefficient + 0″·60 and the colatitude 21″·80 were used in that period, and no general correction to the R – D discordance is yet available.

6. Newcomb's corrections are published for every 5° of declination. For the purpose of this paper a mean correction is needed and for the investigation of the coefficients of sin F, cos F, a correction with the right ascension α as argument. Since there can be no correction involving $\cos\alpha$, and since, if we neglect the Moon's latitude, the declination δ is equal to 0·4 sin α approximately, the correction was assumed to be of the form $a + b \sin\alpha$. The values of α for every 30° corresponding to those of δ at $\delta = 0$, $\pm 12°$, $\pm 20°$, $\pm 23°{\cdot}5$ were used in the analysis in each case, as sufficiently approximate. The mean values adopted have been obtained in this way. The coefficients b are not set down here.

For the R – D discordance depending on sin Z.D., the factor, obtained in the same way, is 0·75, and the adopted coefficient has been multiplied by this factor for observations of the Moon.

7. The adopted corrections are shown in the following table.

TABLE I.

Corrections to the observed N.P.D. of the Moon. Units of 0″·01.

Year.	0	+10	+20	+30	+40	+50
1851	−49	−15	+47	−48	−29	−22
1852	−49	−42	+51	−41	−32	− 7
1853	−33	−40	+50	−45	−42	− 7
1854	−50	−45	+51	−45	−30	−17
1855	−54	−51	+57	−47	−19	−11
1856	−48	− 7	+55	−50	−16	0
1857	−49	− 5	+57	−47	+ 5	0
1858	−82	+61	−15	−46	+ 5	0
1859	−61	+63	−19	−40	− 4	0
1860	−55	+66	−25	−31	−12	0

1847–51, −24; 1911, 0.

These corrections to the north polar distances are with sufficient accuracy considered also as corrections to the mean ecliptic north polar distances, and are included in the fourth column of Table II.

8. *Corrections for Right Ascension Error.*—In the greater part of the period the right ascensions require corrections of the form $a \sin\alpha + b \cos\alpha$. The latitude of the Moon will therefore require a mean correction $-\frac{1}{2}b \sin\epsilon = -\frac{1}{5}b$. This is very small, but, since it had already been obtained for the investigation of the eccentricity and perigee, it is inserted here. The numerical values are given by Newcomb on p. 160 of his Fundamental Catalogue. They give the following corrections to the observed E.N.P.D. of the Moon:—

1847, 1848	+0″·01	1862–69	+0″·06
1849–55	+0 ·05	1870–77	+0 ·03
1856–61	+0 ·03	1878–88	+0 ·02

These corrections are included in the fourth column of Table II.

9. *Corrections for Parallax.*—The constant of sine parallax used in reducing observations of the Moon to the Earth's centre has been fully investigated by Newcomb in his last paper * on the occultations. He obtains for its value, from theoretical considerations,

$$3418''\cdot65\{1 + c(\tfrac{1}{3} - \sin^2\phi)\},$$

where c is the compression and ϕ the latitude of the place of observation. I adopt the compression 1/297. This, with $\phi = 51^\circ\ 28'\ 38''$, gives for the adopted sine parallax at Greenwich

$$3415''\cdot44,$$

which differs very little from the last Greenwich-Cape determination† by observation. For the purpose of this paper the theoretical value is to be preferred.

The residuals from 1847–61 were taken from the comparisons given in the Appendix to vol. l. of the *Monthly Notices*. The observations had been corrected to Adams's value, which is $3422''\cdot33\ (1 - c_1 \sin^2\phi)$. The compression $c_1 = 1/300$ was used, so that the Greenwich constant from 1847–61 was

$$3415''\cdot35.$$

Since 1862 Hansen's value has been used with the compression 1/300. It is, at Greenwich—

$$3422''\cdot07(1 - \tfrac{1}{300}\sin^2\phi) = 3415''\cdot09.$$

The correction to the latitude of the Moon is, with sufficient accuracy,

$$\delta\beta = \delta\pi\,.\,\sin(\phi - \delta) = \delta\pi(\sin\phi - 0\cdot4\,\cos\phi\,\sin a),$$

where $\delta\pi$ is the correction to the parallax for Greenwich.

Hence the corrections to be applied to the observed N.P.D. are

$$\begin{aligned} 1847\text{–}1861, &\quad -0''\cdot07 + 0''\cdot02\,\sin a, \\ 1862\text{–}1911, &\quad -0\cdot28 + 0\cdot09\,\sin a. \end{aligned}$$

The periodic term is not needed here. The constant parts are included in the fourth column of Table II.

10. *The Mean Correction.*—Hansen adds $-1''\cdot00$ to all his tabular latitudes. After all corrections had been made, including the addition of $+1''\cdot00$ in order to cancel Hansen's addition, a mean tabular minus observed correction of $+0''\cdot51$ was obtained. Neither of these is inserted in the fourth column of Table II., but only in the fifth column, where $+1''\cdot00 - 0''\cdot51 = +0''\cdot49$ has been added, reducing the adopted mean of the numbers in that column to zero.

11. The fourth column of Table II. therefore contains the sum

* *Amer. Ephem. Papers*, vol. ix. part i. p. 43.
† *M.N.*, vol. lxxi. p. 540.

of the three corrections (with the same signs, since we change from observed north polar distances to tabular – observed latitudes) interpolated for the adopted periods of 414 days. The headings sufficiently explain the other columns. It may be added that one period is equal to 1·1335 years, and that the middle of period 86 is the date 1847·64.

TABLE II.

Mean Latitude. Tabular – Observed. Units of 0″·01.

Per.	Obs.	Tab. Cor.	Obs. Cor.	Sum +49.	Per.	Obs.	Tab. Cor.	Obs. Cor.	Sum +49.
86	– 63	– 7	–30	–51	115	–28	+ 2	–54	–31
87	– 52	+13	–29	–19	116	+56	–18	–75	+12
88	– 79	+17	–26	–39	117	+79	–22	–69	+37
89	– 20	– 1	–40	–12	118	+64	– 5	–73	+35
90	+ 14	–18	–52	– 7	119	+48	+15	–74	+38
91	– 5	–14	–39	– 9	120	+13	+15	–77	0
92	– 7	+ 4	–51	– 5	121	+38	– 2	–72	+13
93	– 6	+12	–56	– 1	122	+77	–13	–72	+41
94	– 2	+ 1	–53	– 5	123	+20	– 3	–67	– 1
95	+ 11	–11	–65	–16	124	+48	+14	–59	+52
96	+ 25	– 5	–77	– 8	125	+ 4	+12	–58	+ 7
97	+ 20	+14	–62	+21	126	+23	– 6	–65	+ 1
98	– 32	+22	–32	+ 7	127	+53	–17	–63	+22
99	+ 3	+ 5	–57	0	128	+51	– 5	–51	+44
100	+ 14	–19	–64	–20	129	–10	+15	–45	+ 9
101	+ 33	–22	–68	– 8	130	–41	+16	–24	0
102	+ 28	– 3	–61	+13	131	– 7	– 3	–25	+14
103	– 30	+15	–29	+ 5	132	+10	–22	–34	+ 3
104	– 72	+11	+ 4	– 8	133	+22	–17	–44	+10
105	–111	– 5	+40	–27	134	+ 2	+ 4	–43	+12
106	– 90	–14	+40	–15	135	– 5	+19	–35	+28
107	– 97	– 4	+23	–29	136	–22	+12	–43	– 4
108	– 77	+10	+25	+ 7	137	+14	– 4	–39	+20
109	–101	+10	+24	–18	138	–40	– 7	–28	–26
110	– 84	0	+27	– 8	139	–37	+ 2	–28	–14
111	– 84	– 9	+30	–14	140	–42	+11	–28	–10
112	–142	– 3	+30	–66	141	–29	+ 4	–28	– 4
113	– 59	+11	–21	–20	142	– 9	–11	–28	+ 1
114	– 54	+15	–46	–36					

12. *The Final Residuals.*—It is recalled that the mean of the residuals in the last column of Table II. is zero, the actual mean being +0″·51 after all observational and tabular corrections have been taken into account. This mean, which had been subtracted,

is replaced in the following discussion, and becomes here an observed error of the E.N.P.D.

A glance at the residuals shows some fairly well defined groups between which the residuals seem to change their average values. The periods 86–89, in which the mural circle was used, also form a separate group on account of possible instrumental errors. These groups with their approximate dates and mean errors are as follows:—

TABLE III.

Group Results from Meridian Observations of the Moon.

Periods.	Years.	Mean Value.	Diff. from General Mean.	Mean Error.
		″	″	″
86–89	1847·1 – 51·6	+0·21	−0·30	0·39
90–103	1851·6 – 67·5	+0·49	−0·02	0·11
104–115	1867·5 – 81·1	+0·29	−0·22	0·29
116–129	1881·1 – 97·0	+0·73	+0·22	0·29
130–137	1897·0 – 05·0	+0·61	+0·10	0·11
138–142	1905·0 – 11·7	+0·40	−0·11	0·13

The final values deduced directly from Table II. are:—

Mean value of E.N.P.D.	+0″·51 + 90°
Probable error of result	±0 ·021
,, ,, one period	±0 ·16
,, ,, one observation	±1 ·8

If we exclude the periods 86–89, the mean is 0″·496, the inclusion of them gives 0″·518; the adopted mean is obtained by assigning to them half-weight.

13. *Comparison with Newcomb's Result.*—In his last paper, referred to in No. 9, Newcomb finds for the mean residual, from occultations, +0″·36 ± 0″·14. In this result, however, he has combined the immersions and emersions, and the former include occultations back to 1753. If we take the immersions alone during the last seventy years, the result would lie between 0″·50 and 0″·55 (with a probable error of less than 0″·04) according to the method of weighting the observations. The emersions give between 0″·10 and 0″·20.

14. *Observations of the Sun and of Polaris.*—The colatitude as deduced at Greenwich each year from observations of the Sun is found in the annual volumes. The results to 1879 are collected in a paper by Christie,* and those from 1887 to 1905 in the Greenwich Catalogues of 1890, 1900. I have applied to them exactly the same corrections as those applied to the Moon (with, of course, the exception of the parallax correction), so that the following table of residuals, reduced to colatitude 21″·80, should be strictly comparable with those obtained in Table II.

* *Mem. R.A.S.*, vol. xlv. pp. 156, 157.

TABLE IV.

Residuals of the Sun's E.N.P.D. above 90°. Units of 0″·01.

Year.	+0	+10	+20	+30	+40	+50	+60
1851	− 1	+48	−40	−49	−22	− 5	−15
1852	− 1	+29	−21	−77	−52	+ 5	
1853	+ 4	+35	−31	−21	−37	+14	
1854	+ 3	+59	−33	+ 7	+ 2	−14	
1855	+ 9	+ 6	−34	0	−33	+ 3	
1856	−14	− 1	−19	−35	−15	+ 1	
1857	+18	−11	−53	−59	+ 1	−10	
1858	−11	− 9	−44	−53	+ 5	+ 1	
1859	+10	+20	−59	−46	− 4	− 2	
1860	−10	−19	−20	−21	− 4	−16	

Some minute tabular corrections should possibly be made to the residuals before 1864, when Leverrier's solar tables replaced those of Carlini, but the differences could hardly amount to more than a few hundredths of a second of arc in the annual means. The change from Leverrier's solar tables to those of Newcomb in 1901 makes no difference to the mean latitude, since the latter used Leverrier's perturbations for this co-ordinate.

The annual results for Polaris up to 1893 are given by W. G. Thackeray in a discussion of the constant of nutation.* They have been corrected by him to the refractions of the Fundamenta and colatitude 21″·90 and for an assumed proper motion; but as the observations are only quoted here for the purpose of showing how the errors of observation compare with those of the Sun and Moon, an error of the form $a + bt$ plays no part in the discussion.

The next table shows, in five-year groups, the variations in the results obtained from the meridian observations of the Moon, of the Sun, and of Polaris. To those of the Moon the constant −0″·51 has been added, to those of the Sun the constant 0″·12, and to the results for Polaris a constant to make them approximately comparable with those of the Sun. The residuals for Polaris are those of N.P.D. and those for the Sun and Moon of E.N.P.D. The results for the Moon have been obtained (by interpolation where necessary) from Table II.

TABLE V.

Residuals in Five-Year Groups of Polaris, Sun, and Moon. Units of 0″·01.

	Year 1800+	53·5.	58·5	63·5.	68·5.	73·5.	78·5.
Polaris .	. . .	0	+11	+10	+10	− 6	− 1
Sun + 12	. . .	+15	+11	+47	+ 8	−20	−27
Moon − 51	. . .	− 6	− 1	− 1	−11	−12	−36
	Year 1800+	83·5.	88·5.	93·5.	98·5.	103·5.	108·5
Polaris .	. . .	− 6	−27	− 4*			
Sun + 12	. . .	−16	−31	−16	+ 9	+13	+ 7
Moon − 51	. . .	+27	+21	+18	+ 7	+ 5	−12

* Three years only.

* *Mem. R.A.S.*, vol. li. p. 263.

15. Finally, in order that direct comparison may be made between the results for the Sun, the Moon from Greenwich meridian observations and from the occultations, immersions and emersions, I have arranged them in groups to correspond with those of Newcomb. In this table the constants of the Sun and Moon have been restored, so that the residuals are the direct results of the observations corrected to colatitude 21″·80.

TABLE VI.

Group Residuals of Observations of E.N.P.D. of the Sun and Moon. Units of 0″·01.

	Years	1857–73.	1874–90.	1891–08.
Sun		+ 4	− 26	− 10
Moon, mer. obs. . . .		+42	+51	+54
,, immersions . .		+62	+52	+48
,, emersions . . .		0	+17	+19

Discussion of the Results.

16. *Errors of Observation.*—The group arrangements in Tables III., V., VI. show the principal changes which have taken place in the observed latitude of the Moon during the past sixty-five years. Tabular errors may be excluded altogether. Although the residuals have not been corrected for the change of mean longitude introduced in 1883, this should have little effect on the means over periods of 414 days. Moreover, practically all inequalities due to forces other than that of the Sun's attraction produce short-period (a month) inequalities in the latitude, so that there is no reason to fear that possible omissions can have any influence. We can therefore treat the residuals almost certainly as errors of observation. Their nature indicates the same cause. The changes from group to group are sudden: the largest, 0″·44, between the third and fourth groups of Table III., took place between 1880 and 1882, as a reference to Table II. shows. To the last group no observational corrections have been made.

Table V. has been formed to investigate the source of this difference. It will be noticed that for Polaris the change is − 0″·05, for the Sun + 0″·11, and for the Moon + 0″·63 between 1878·5 and 1883·5; the change can therefore hardly be instrumental. The same argument must, I think, apply to the changes in Polaris and the Sun as compared with those of the Moon from 1868·5 to 1873·5. In fact, in no case do the observations of all three bodies indicate instrumental errors of the same kind amounting to as much as 0″·1; where two of them show a marked change, that in the third is either quite small or in the opposite direction.

The changes in the adopted coefficient of the R − D discordance at 1878 and 1882 require notice. A change tending to improve the lunar observations will create greater discordance in those of the Sun; in fact, the discordances in the mean of the two are comparatively small except between 1870 and 1880. These could be improved by the assumption of a gradual change

from 0″·30 in 1870 to 0″·68 in 1883 instead of the adopted sudden changes of +0″·21 in 1878 and +0″·17 in 1883.

In this connection the observations from 1897 to 1905, the period included in the Greenwich Catalogue of 1900, are interesting. In the following table I have interpolated those of the Sun from Table IV. to make them correspond with the periods of 414 days used for the Moon; 0″·61 (not 0″·51) is subtracted from those of the Moon (Table II.) in order to show the variations clearly.

TABLE VII.

Corrected Residuals of the Sun and Moon from 1897–1905. *Units of* 0″·01.

Per.	Sun.	Moon.	Per.	Sun.	Moon.
130	+1	−10	134	0	+ 2
131	+3	+ 4	135	+11	+18
132	−4	− 7	136	− 8	−14
133	−4	0	137	+ 2	+10

Some of the variations of the Sun are rather less than the actual changes owing to the interpolations. It is rather remarkable that they should follow those of the Moon so closely. It was in this period that the observations for the R − D discordance required large corrections to reduce them to the adopted value. The observations seem to indicate some small error of observation common to both. I have not found any such correspondence in the periods immediately preceding or following this decade.

The most probable supposition to account for the majority of the changes appears to be that of personal equation. The memoir (*loc. cit.*) by W. G. Thackeray* is of assistance here. He has shown (p. 261) that in the case of the right ascensions of Polaris, which show a break about 1870 similar to that of the north polar distances, a reduction to one observer of the errors of personal equation eliminates a large part of the difference, and it is reasonable to suppose that similar results might have been obtained in the north polar distances of both Polaris and the Sun. The various changes in the difference between the results for the Sun and Moon shown in Table V. indicate this solution; the means of the two in the five-year groups from 1880 to 1911 are all less than 0″·1. But I must leave the investigation of this question aside.

17. Table VI. shows the results obtained from the meridian observations of the Sun and Moon and from the occultations in the last fifty years. It is seen at once that the meridian observations and the immersions give practically the same result, while the emersions show a residual 0″·4 smaller. But in this connection the different methods of observation must be taken into account. The occultations depend on the declinations of Newcomb's Fundamental Catalogue; those at Greenwich depend only on the measured distances of the Moon from the zenith and on an adopted colatitude. In order to see what Newcomb's result would have

* *Mem. R.A.S.*, vol. li.

been if he had adopted the Greenwich declinations of the stars which presumably possess the same instrumental errors as those of the Moon, I compared the mean values of the corrections adopted here with those of the Greenwich catalogues as given on pp. 233–6 of Newcomb's Fundamental Catalogue and in the Greenwich Catalogues of 1890, 1900. The mean difference came out to be less than 0″·03. We may therefore conclude that practically the same declination system is used in both cases.

Although the emersions, from the nature of the observations, possess a much lower degree of accuracy than the immersions, there does not seem to be any reason due to this cause why the mean latitude should differ systematically. I conclude that the difference is not due to errors of observation; it is discussed below.

The general conclusion at which I have arrived is that the mean residuals obtained from the Greenwich observations, as well as those from the immersions and emersions, are real and are not to be explained by instrumental or other errors of observation. They amount to +0″·5 for the meridian observations and for the immersions and to +0″·1 for the emersions.

18. *Possible Sources of the Difference.*—Various suppositions may be made as to the cause of the apparent position of the mean orbit of the Moon below the ecliptic.

(*a*) An error of the parallax of the Moon. The theoretical and observed values agree so nearly (see No. 9) that any probable change could only account for a small part of the residual.

(*b*) An error in the adopted colatitude. The observations of the Sun and Polaris and numerous other determinations show that the error can hardly be greater than 0″·1. Further, Newcomb's occultations depend on observations at many observatories; such a supposition would involve a general error. In this connection we need only to consider the difference between observations of the Sun and those of the Moon in order to reject the hypothesis.

(*c*) A difference between the centre of mass and centre of figure of the Moon. This was Hansen's original supposition to account for the 1″·00 which he subtracted from all latitudes. Newcomb (*loc. cit.* in No. 9) inclined to the same explanation. An equally possible explanation is a difference between the same two points in the case of the Earth. There does not seem to be any astronomical method of deciding between the two hypotheses; the parallax enters only with the factor $\sin(\phi - \delta)$, and an increase of ϕ produces the same effect as an equal decrease of δ. There are grave objections to the latter hypothesis. From a geological point of view, isostatic compensation would be seriously affected by a difference of over a third of a mile between the two points, though this would apply with greater force to the Earth than to the Moon. (It is not impossible, however, that annual changes due to the deposition of snow might be detected by means of occultations or the Harvard photographic method for finding the Moon's position.)

(*d*) Local irregularities of the Moon's limb. In this connection the emersions (see Table VI.) should also be considered. The

combination of the meridian observations with the occultations requires a considerable area of depression of the whole north-east limb, extending over the North Pole and to a much smaller degree down to the western end of the Equator; similar areas of elevation in the southern halves of the limbs will have the same effect. This hypothesis is therefore not essentially different from the previous one. In this form it is much more plausible on account of the known measured irregularities of the Moon's limb. It might seem that measures of a crater like Mösting A should be free from such sources of error, but the reductions to the centre of the Moon will implicitly contain, at least in part, the effects of such large areas. Ultimately it seems probable that the question may be settled by the comparison of these measures with the meridian observations and with occultations, and perhaps even more certainly by the photographs now being taken at Harvard and measured and discussed by Professor H. N. Russell at Princeton. In the short series already published,* comprising four nights of Harvard photographs and ten of Greenwich observations during the same month, the correction $0''{\cdot}6$ was obtained in both cases. Subtracting $0''{\cdot}4$ for parallax and tabular corrections of long period, a residual of $0''{\cdot}2$ is left. Nothing more can be said concerning this than to point out the fact that the residual is of the proper sign; short-period tabular corrections may easily account for the difference between this value and that obtained above or may reduce it to zero.

Whatever be the form of the hypothesis, it seems certain that all meridian observations of the mean of the north and south limbs will require a correction of about $0''{\cdot}5$. In the longitude there is a well-marked residual with period and phase very nearly, if not exactly, equal to those of the Moon's longitude. After making several tabular and observational corrections, I find (in a paper to be published shortly) a coefficient $0''{\cdot}4$ for this term. This can be explained by the same hypothesis. [Note added 1914 January 6. Since the above was written a similar term has been found in the latitude, still further strengthening the hypothesis. I shall discuss the question more fully in a paper to appear shortly.]

The hypothesis of systematic irregularities of the limb is confirmed qualitatively by the papers of Franz† and Hayn.‡ The map shown by the former would, if the high and low areas there depicted extend to the limbs, give corrections similar to those described above, while the outline of the limbs, exhibited by the latter, appears to afford the same result. It is possible that the results in these papers, combined with those obtained above, will furnish the proper correction to be made for all observations in declination.

* See *Annals of Harv. Coll. Obs.*, vol. lxxii. No. 1.
† *Königsberger astr. Beob.*, Abth. 38.
‡ *Abh. der Königl. Sächs. Ges. der Wis. zu Leipzig*, Bd. xxx.

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The Proper Motions of the Brighter Stars within 17° of the Pole considered in relation to their Spectral Type. By H. Spencer Jones, B.A., B.Sc. (Plates 6, 7.)

(Communicated by the Astronomer Royal.)

In the *Astrophysical Journal* for 1912 October there was published a paper by J. A. Parkhurst, entitled "Yerkes Actinometry." In this paper were given the results of the determinations of (1) the photographic magnitudes; (2) the "visual" magnitudes (with colour-sensitive plates, and "visual-luminosity" filter); (3) the resulting "colour-indices"; and (4) the spectral types, according to the Harvard system of classification, of 672 of the brighter stars between zones 73° and 90° of north declination. It thus contains the spectral types of a large number of stars in a comparatively small region of the sky, which reach a lower limit of magnitude than the stars which have been classified at Harvard. The visual magnitudes, in fact, range (with one or two exceptions) from about $4^{m}\cdot0$ to $8^{m}\cdot5$, the mean being $6^{m}\cdot8$. It seemed that if the proper motions of these stars were determined, there were prospects of obtaining some valuable results from a discussion of their values in relation to the spectral type. Moreover, the new *Greenwich Astrographic Catalogue* (not yet published) gives the photographic position for the epoch 1900 of all the stars which are contained in Parkhurst's Catalogue, and so offered fresh and valuable material for the determination of many of the proper motions.

Lewis Boss* has discussed very thoroughly, for more than 4000 of the brightest stars of the sky, the relations between the proper motions and spectral types, and has determined the average parallaxes and velocities of the various types. The relationship between the average velocities and spectral types is very clearly shown. In obtaining his results he excluded all stars with a proper motion of over 20″ per century: little justification can be offered for this proceeding, because the majority of the stars of large proper motion belong to classes F and G, and so the average parallaxes of these classes have been unduly reduced as compared with the other classes. Campbell† has also discussed the Lick Observatory radial velocity determinations in their relation to the spectral types. These investigations by Boss, Campbell, and by others have been concerned with bright stars scattered all over the sky. No previous discussion has beeen made for a number of comparatively faint stars distributed over a small region of the sky.

About one-quarter of the stars dealt with in this paper are contained in Boss's *Preliminary General Catalogue*, and for these the proper motions there given have been used. Of the remainder, 84 are contained in Carrington's Red-Hill Catalogue, for which proper motions have been determined by comparison with the *Greenwich Astrographic Catalogue*, as explained in a recent

* *Astron. Journal*, vol. xxvi., Nos. 623–4.

† *Lick Obs. Bulletin*, vol. vi., No. 196, p. 125.

paper by the Astronomer Royal * on "The Distribution in Space of the Stars in Carrington's Circumpolar Catalogue," and these proper motions have been used in the present work. I have specially determined the proper motions of the remaining stars. The determinations have been based upon the observations in the following catalogues:—

1. Groombridge's Catalogue, epoch 1810;
2. The First Radcliffe Catalogue, epoch 1845;
3. Oeltzen's reduction of Schwerd's *Beobachtungen von Circumpolarsternen*, epoch 1828;
4. The Kasan and Dorpat Sections of the A.G.C. zones, epoch 1875;
5. *The Bonn Observations*, vol. vi., epoch 1855;
6. Oeltzen's reduction of Argelander's Northern Zones, epoch 1842;
7. Baily's reduction of Lalande's observations in the *Histoire Céleste*, epoch 1800; and

7A. Federenko's *Lalande*, with supplement, epoch 1790.

In a few cases Piazzi's Catalogue has also been used.

For any given star, values of the proper motion have been derived by comparing its positions as given in some or all of these catalogues with the modern Greenwich photographic position, epoch 1900. These latter positions have been used as a basis throughout. Thus, in some cases, as many as ten separate determinations of the proper motion of a star have been obtained: in other cases there are only one or two determinations, and for these the value of the proper motion adopted may be considerably in error. These weak determinations are indicated in the catalogue; their number is not large. In reducing the star positions to obtain the proper motions the systematic corrections in right ascension and declination, as given by Boss, have been applied to the observations in catalogues 1, 2, 3, 4. Zone corrections were applied to the observations of Argelander: these were determined by reducing all the Boss stars in the zones used to the epoch 1842, using Boss's proper motions, and comparing the reduced position with the position given in the O.A. The mean of the differences for all the stars in a given zone common to the two catalogues gave the zone correction. The same process was repeated for the stars common to Groombridge and O.A., using the proper motions given in the "New Reduction of Groombridge's Catalogue" by Dyson and Thackeray, and applying Boss's systematic corrections to the Groombridge positions. A weighted mean of the two separate determinations was adopted as the final zone correction. Throughout the reductions for the determination of the proper motions Newcomb's value of the precession has been used.

In taking the weighted means of the various proper-motion determinations, the weights assigned by Boss to the catalogue observations have been used. In the other cases the probable error of a catalogue position was calculated from comparisons of

* *M.N., R.A.S.*, vol. lxxiii., 1913, p. 336.

the various observations of stars observed two or more times. Since Boss's unit of weight corresponds to a probable error in the catalogue position of $\pm''\cdot30$, weights on Boss's scale could be derived for these catalogues by using the relation $w\epsilon^2 = w_0{\epsilon_0}^2 = (\cdot30)^2$, w being the weight corresponding to a probable error of amount ϵ. The final weights adopted in taking the weighted mean were proportional to wt^2, w being the weight of the observation, and t being the difference between the mean epochs of the Greenwich and catalogue observations.

The results of the determinations are given in the following catalogue, which contains all the stars given by Parkhurst except two: one the faint component of the double Σ 2136; and the other β Ursæ Minoris, a star brighter than the others, and for which neither photographic magnitude, colour-index, nor spectral type are given. The stars are arranged in order of increasing right ascension. The first column gives the B.D. number of the star, serving to identify it. An asterisk is placed against the zone number in those cases where precession between 1855 and 1900 has changed the declination to the adjacent degree. The second, third, and fourth columns contain respectively the visual magnitude, the colour-index (defined as the algebraic difference between the photographic and visual magnitudes), and the spectral type as given by Parkhurst. The fifth and sixth columns contain the components of the proper motions in R.A. and declination. These have all been resolved into components towards and perpendicular to the solar apex, which was assumed to have the position R.A. 270°, Dec. +34°: these resolved components are given in the seventh and eighth columns. The apical component v has been given the positive or negative sign according as the proper motion is towards or away from the antapex. The positive direction of the cross motion τ is obtained by a right-handed screw rotation of 90° about the line from the Sun to the star, from the positive direction of the apical motion. The ninth column gives the distance λ of the star from the assumed apex. The proper-motion determinations which are weak are indicated in the usual manner by a colon.

Catalogue of Zones +73° to +90°.

No. in B.D.		Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.
73°	2	7·30	+ ·03	A1	+ 0ˢ·63	+ 3″·4	+ 2″·0	+ 4″·1	58°
*78	1	5·97	+ ·12	A3	+ 3·63	− 2·9	+11·6	+ 1·0	57
73	4	7·05	+1·11	G±	+ 1·66	− 2·5	+ 7·0	− 1·2	57
75	4	7·21	+ ·91	G±	+ 0·89	+ 0·5	+ 3·1	+ 1·1	58
76	5	6·46	− ·25	A0	+ 0·68	− 0·1	+ 2·4	+ 0·4	58
75	7	7·15	− ·16	A0	+ 0·99	0·0	+ 3·6	+ 0·8	58
79	10	6·40	− ·20	B8	+ 0·81	+ 0·5	+ 1·8	+ 0·8	58

Catalogue of Zones +73° to +90°—continued.

No. in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
76° 10	6·18	+ ·87	G4	+ 9ˢ·54	− 1″·6	+32″·7	+ 6″·9	58°	
81 13	6·25	+ ·62	F5	− 5·23	+ 9·1	−12·8	+ 6·2	57	
76 14	6·64	+1·21	K0	+ 1·12	+ 1·6	+ 3·3	+ 2·7	59	
75 36	7·12	+ ·72	G0	+10·34	− 9·8	+40·3	+ 3·5	60	
74 27	5·67	+ ·21	A1	− 0·53	− 2·4	− 1·1	− 3·0	60	
74 29	5·51	− ·12	A0	+ 0·44	− 1·2	+ 2·1	− 0·5	60	
77 25	6·91	+1·14	G3	+ 0·71	− 3·2	+ 3·1	− 2·3	59	:
77 27	6·59	+ ·31	A7	+ 2·18	0·0	+ 6·6	+ 2·3	59	
*72 43	7·69	+ ·89	G±	− 0·22	− 4·4	− 3·6	− 2·6	61	
*82 20	5·50	+ ·12	A1	+ 3·1	− 1·0	+ 5·7	+ 0·6	58	
73 40	7·55	+ ·20	A5	+ 0·66	+ 0·5	+ 1·3	+ 1·0	61	
*79 24	6·64	+ ·34	F0	+ 4·09	+ 2·1	+ 9·3	+ 5·6	59	
*83 20	6·66	+ ·21	A4	+ 4·33	− 0·5	+ 6·5	+ 1·5	57	
85 19	4·25	+1·22	G4	+ 7·7	− 0·5	+ 8·3	+ 2·0	57	
88 4	6·45	− ·02	A1	+18·0	− 1·6	+ 7·2	+ 0·3	56	
73 51	6·73	+ ·02	A1	+ 0·81	− 0·6	+ 3·4	+ 0·8	61	
86 17	6·14	+1·21	G5	+ 5·9	− 1·7	+ 5·6	0·0	57	
79 29	6·17	+ ·99	G2	− 1·14	− 4·0	− 1·4	− 4·9	59	
*78 34	5·59	− ·27	B9	+ 3·24	+ 0·8	+ 8·1	+ 4·3	60	
73 59	7·58	+ ·69	G±	+ 0·05	− 1·0	+ 0·5	− 0·9	62	
79 36	6·18	+ ·41	F1	+ 1·73	+ 6·2	+ 0·5	+ 7·8	60	
73 60	7·86	...	...	+ 0·46	− 2·1	+ 2·7	− 1·0	62	
73 61	6·96	+ ·01	A±	+ 0·57	− 1·9	+ 2·9	− 0·6	62	
80 35	7·14	+ ·41	F0	+ 3·14	+ 1·3	+ 6·8	+ 4·2	59	
80 36	6·58	+ ·05	A1	+ 0·15	− 1·7	+ 1·0	− 1·3	59	
76 39	7·12	+1·25	G − K	− 2·39	− 2·4	+ 8·7	+ 1·5	61	:
*76 40	6·23	+1·00	G4	− 0·29	+ 9·2	− 4·9	+ 7·9	61	
78 36	7·06	+1·51	K5	+ 9·27	− 6·0	+ 9·9	− 1·2	60	:
*73 66	7·12	+ ·17	A4	+ 0·03	− 1·8	+ 0·8	− 1·6	62	
*75 58	7·12	+ ·77	G2	− 0·79	− 6·6	+ 0·5	− 7·2	61	
75 59	6·49	+ ·13	A5	+ 2·18	− 2·5	+ 8·3	+ 1·5	61	
*77 49	5·84	+ ·47	B5?	+ 0·12	+ 0·8	0·0	+ 0·9	60	
73 75	7·15	+ ·59	F8	+ 4·63	−12·5	+23·2	− 1·2	63	
73 76	9·00	...	...	− 0·24	− 3·7	+ 1·0	− 3·6	63	
88 8	2·11	+ ·42	...	+13·8	+ 0·1	+ 4·0	+ 1·7	56	
73 81	6·62	− ·09	A0	+ 0·66	− 1·4	+ 3·1	+ 0·3	63	
80 50	7·01	·00	A0	+ 0·37	+ 1·2	+ 0·2	+ 1·5	60	
77 58	6·64	·00	A0	+ 0·53	− 0·4	+ 1·6	+ 0·5	62	

Catalogue of Zones +73° to +90°—continued.

No. in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
				s	″	″	″	Å	
*73° 92	6·55	+ ·97	G3	+ 1·46	− 2·6	+ 6·4	+ 1·1	63	
75 72	7·18	+ ·19	A4	+ 0·19	− 2·0	+ 1·6	− 1·3	63	
80 55	6·86	+ ·08	A0	− 0·07	− 0·7	+ 0·2	− 0·7	60	
80 57	7·35	+ ·17	A0	+ 1·71	− 0·7	+ 3·9	+ 1·5	60	
*74 84	6·86	+ ·39	F4	+ 2·04	− 3·4	+ 8·5	+ 1·5	63	
81 61	6·70	+1·89	K5	+ 1·82	− 0·1	+ 3·5	+ 2·0	60	
80 58	7·10	+1·38	K0	+ 4·71	− 9·1	+14·9	− 1·3	61	
77 65	6·47	+1·28	K0	+ 2·24	− 3·3	+ 7·8	+ 1·0	62	
75 76	6·88	+ ·32	A5	− 2·15	− 1·1	− 6·2	− 5·5	63	
74 87	6·59	+ ·93	G2	+ 0·02	− 2·6	+ 1·6	− 2·0	64	
75 83	6·96	+1·66	K±	− 0·15	+ 0·7	− 0·9	+ 0·2	64	:
77 73	5·92	+1·31	G9	+ 0·26	0·0	+ 0·7	+ 0·5	63	
*74 91	6·59	− ·11	A0	+ 0·59	− 1·2	+ 2·6	+ 0·4	64	
73 108	6·11	+ ·21	A3	− 1·04	+ 0·1	− 3·6	− 2·6	65	
76 63	5·52	+ ·06	A5	+ 3·87	− 5·4	+14·0	+ 3·5	63	
*79 61	7·51	+ ·13	A2	+ 1·43	− 2·4	+ 4·4	0·0	61	
75 86	5·45	+ ·79	G2	− 0·20	− 1·6	+ 0·4	− 1·8	64	
*73 112	7·40	+ ·13	A1	0·00	− 1·2	+ 0·7	− 1·0	65	
80 64	5·95	+ ·08	A2	− 0·91	+ 0·9	− 2·3	− 0·5	61	
74 95	7·58	+ ·01	A2	+ 1·17	− 2·7	+ 5·4	+ 0·8	64	
78 69	6·98	+ ·34	A6	+ 0·07	+ 0·2	+ 0·0	+ 0·3	62	
*80 65	6·77	+ ·19	A4	+ 2·26	− 1·2	+ 5·0	+ 1·9	61	
*78 71	7·19	...	A2	0·00	+ 5·6	− 3·2	+ 4·4	62	
79 63	6·45	...	A2	+ 0·05	+ 3·4	− 1·9	+ 2·8	62	
*82 51	6·36	+ ·92	G2	+ 3·7	− 4·1	+ 7·9	+ 0.4	60	
73 121	6·18	+ ·98	G3	+ 1·43	− 3·7	+ 7·0	+ 1·0	65	
78 73	6·99	+ ·07	A1	− 0·34	+ 0·6	− 1·2	− 0·1	62	
75 94	7·55	+ ·05	A0	+ 0·84	− 2·3	+ 3·9	+ 0·3	65	:
*72 128	8·11	+ ·08	A±	+ 0·06	− 2·2	+ 1·7	− 1·4	66	
73 139	7·18	+ ·36	A7	+ 1·23	− 2·9	+ 5·8	+ 1·3	66	
77 84	7·68	+ ·68	G±	+ 0·28	+10·4	− 6·8	+ 7·8	64	
79 75	7·40	+1·05	G2	+ 1·05	− 2·6	+ 3·9	0·0	63	
76 81	6·91	+1·64	K±	+ 2·61	− 3·8	+ 9·8	+ 3·8	64	
*74 111	7·94	+ ·20	A−K±	− 0·63	− 2·1	− 0·3	− 3·2	66	
74 112	8·20	+ ·26	...	− 0·41	− 2·5	+ 0·6	− 2·9	66	
*80 86	5·67	+1·34	K2p	+ 0·89	− 6·9	+ 6·2	− 3·6	62	
74 117	7·13	+ ·17	A3	− 0·78	− 3·6	+ 0·5	− 4·7	66	
79 86	7·25	+ ·02	A2	+ 0·18	+ 0·5	0·0	+ 0·7	63	

Catalogue of Zones +73° to +90°—continued.

No. in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
				s	″	″	″	°	
74° 120	7·57	+ ·17	A4	+ 0·98	− 3·5	+ 5·3	+ 0·5	67	
*75 109	7·42	+1·15	G5	+ 0·34	− 0·2	+ 0·9	+ 0·7	66	:
76 101	6·90	+ ·88	G2	− 0·46	− 0·7	− 0·5	− 1·6	65	:
*78 103	5·38	+1·87	M	− 1·30	+ 1·1	− 3·2	− 2·0	64	
74 131	7·08	− ·15	B8	− 0·32	− 3·1	+ 1·5	− 3·0	67	
73 165	7·34	+ ·10	A2	− 0·11	+ 0·4	− 0·6	− 0·1	68	:
*80 97	5·71	+ ·16	A4	− 2·00	+ 0·2	− 3·3	− 3·3	62	
75 124	7·07	+1·44	K±	+ 0·11	+ 0·5	− 0·1	+ 0·6	67	:
75 127	7·48	+ ·35	A − F	− 0·71	+ 2·8	− 3·8	− 0·2	67	
77 109	7·13	+1·48	K±	− 0·48	+ 0·9	− 1·6	− 0·5	65	
*73 168	4·82	− ·36	A0	− 0·70	− 8·3	+ 6·4	− 5·2	68	
79 94	6·59	+1·09	G4	− 0·38	− 5·7	− 1·1	− 5·7	63	
73 170	7·04	+ ·62	F2	+ 0·37	− 2·5	+ 2·9	− 0·4	68	
73 171	6·74	+1·65	G − K±	+ 0·10	− 1·4	+ 1·4	− 0·5	68	
*77 111	7·33	+1·29	K0	+ 1·28	− 2·3	+ 4·3	+ 1·6	65	
78 109	6·74	+ ·15	A3	− 0·14	+ 0·6	− 0·7	0·0	64	
73 172	6·45	+1·52	K+	− 0·06	− 3·8	+ 2·8	− 2·6	68	
81 107	6·82	+1·60	K5	+ 0·12	− 1·1	+ 1·0	− 0·4	62	
77 115	5·45	− ·18	A3	+ 2·03	− 4·9	+ 8·0	+ 2·3	66	
*82 82	7·04	+1·54	K5	+ 0·28	− 1·8	+ 1·7	− 0·8	61	
74 144	7·23	+ ·01	A1	+ 0·34	− 1·6	+ 2·1	+ 0·1	67	
84 59	5·45	+ ·93	G0	+ 5·1	−13·2	+14·9	− 2·9	60	
73 179	6·66	+1·43	G − K±	+ 0·28	− 1·9	+ 2·3	− 0·2	69	
73 180	6·74	+ ·46	F1	+ 5·00	−14·7	+24·1	+ 8·3	68	
*77 119	7·47	+ ·69	F5	+ 1·18	− 2·0	+ 3·7	+ 1·6	65	
74 151	7·87	+ ·11	A3	− 0·12	− 2·1	+ 1·5	− 1·6	68	
77 123	7·13	+1·45	K0	− 0·10	0·0	− 0·2	− 0·2	65	
*72 178	6·40	+ ·05	A1	+ 0·54	− 2·6	+ 3·4	+ 0·5	70	
74 161	7·33	+ ·91	G2	+ 0·50	+ 5·6	− 3·6	+ 4·7	69	
75 143	6·10	+1·05	G3	+ 0·05	+ 0·7	− 0·5	+ 0·5	68	
76 128	7·68	+ ·19	A1	− 0·10	− 4·8	+ 3·8	− 2·9	67	
74 168	6·57	+1·09	G5	+ 0·43	− 1·7	+ 2·4	+ 0·6	69	
83 91	7·58	− ·33	B7	+ 0·45	− 1·6	+ 1·8	0·0	61	
86 51	5·78	+ ·35	F0	+16·4	− 6·6	+14·4	+ 9·0	59	
77 133	6·90	− ·02	A0	+ 0·79	− 1·1	+ 2·2	+ 1·5	66	
74 174	7·62	+ ·94	G±	− 0·01	− 4·3	+ 3·7	− 2·1	69	
73 204	6·58	+1·58	K0	+ 0·10	+ 1·2	− 0·9	+ 1·0	70	
77 138	6·70	+1·25	G5	+ 0·47	− 3·5	+ 3·8	− 0·4	66	

Catalogue of Zones +73° to +90°—continued.

No.	in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	υ.	τ.	λ.
	°				s	″	″	″	°
*75	160	7·61	+ ·33	A8	+ 2·52	− 6·0	+ 9·6	+ 5·3	68
80	125	4·93	+ ·51	F4	− 0·57	− 0·5	− 0·8	− 0·7	64
78	142	...	...	A0	+ 1·06	− 0·2	+ 1·6	+ 2·6	66
74	184	7·01	+ ·43	F2	+ 4·83	−28·6	+34·0	+ 3·6	69
74	186	6·56	+1·10	G5	+ 0·13	− 4·7	+ 4·2	− 2·1	70
73	210	6·26	+1·14	G5	+ 0·32	− 6·7	+ 6·8	0·0	70
78	146	6·74	+1·35	G7	+ 2·00	+ 1·0	+ 2·0	+ 5·8	66
73	212	6·50	+ ·66	F5	− 0·85	+ 2·4	− 3·6	− 2·1	71
80	127	6·20	+1·15	G4	− 0·40	+ 0·5	− 0·9	− 0·7	64
81	147	7·35	− ·03	A0	− 0·97	− 0·5	− 0·5	− 2·0	63
83	104	5·50	− ·46	B6	− 0·1	+ 1·4	− 1·3	+ 0·4	62
85	63	6·41	+ ·50	F6	+ 1·5	+ 3·3	− 2·0	+ 3·2	60
77	150	6·66	+1·04	G3	+ 0·99	+ 0·7	+ 0·7	+ 3·1	67
*82	113	5·47	+ ·72	G0	− 3·0	+10·9	−12·2	− 0·2	62
75	173	6·58	− ·02	A0	+ 0·88	− 1·7	+ 2·7	+ 2·1	68
84	78	7·13	+1·29	K0	− 0·33	− 3·4	+ 2·8	− 1·9	61
80	133	5·28	+1·29	K2p	+ 0·33	− 1·8	+ 2·0	0·0	64
80	134	7·10	+ ·10	A2	− 0·85	+ 0·6	− 1·4	− 1·6	64
80	140	7·22	+ ·27	A5	− 0·49	+ 2·9	− 3·1	0·0	64
83	114	7·04	+1·05	G2	0·00	− 5·0	+ 4·6	− 2·0	62
78	157	7·16	+ ·32	A6	+ 0·13	− 4·0	+ 3·8	− 1·2	66
79	150	6·51	+ ·05	A1	− 0·59	+ 2·0	− 2·5	− 0·8	66
76	174	6·31	+ ·62	F5	+ 2·55	−14·7	+16·8	+ 3·6	69
75	189	5·90	+ ·32	A4	+ 1·17	−13·1	+13·8	0·0	69
75	193	6·87	+ ·90	G1	+ 2·58	−16·4	+18·6	+ 4·3	70
*80	155	5·06	+1·41	K4	− 0·10	+ 3·1	− 3·0	+ 0·7	64
77	178	7·64	+ ·56	F5	− 0·43	− 1·5	+ 1·1	− 1·6	68
74	229	5·94	+1·75	K5	+ 0·71	+ 4·1	− 3·2	+ 3·8	71
73	264	6·06	+1·80	K5+	+ 0·56	− 1·5	+ 2·0	+ 1·9	71
73	265	6·07	− ·04	A0	+ 0·20	− 2·3	+ 2·4	+ 0·2	71
85	74	6·37	+ ·30	A5	+ 3·0	− 6·7	+ 7·4	+ 1·4	60
75	208	7·08	+ ·11	A2	+ 0·41	− 4·8	+ 5·0	+ 0·4	70
73	274	5·52	− ·39	B8	+ 0·40	− 3·2	+ 3·5	+ 0·9	72
76	190	6·29	− ·02	A0	+ 0·44	− 0·3	+ 0·6	+ 1·5	70
74	238	7·27	+ ·20	A4	+ 0·47	− 4·4	+ 4·7	+ 0·9	71
73	280	5·82	− ·14	A0	+ 0·34	− 3·6	+ 3·8	+ 0·7	72
79	169	5·26	+ ·15	F5	− 2·81	+15·5	−16·8	− 4·5	67
75	220	7·90	+ ·31	F0	+ 0·94	− 2·2	+ 2·8	+ 3·1	70 :

Catalogue of Zones +73° to +90°—continued.

No. in	B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
85°	78	6·64	− ·11	A0	− 2ˢ·50	− 0″·3	− 0″·3	− 2″·9	60°	
78	183	6·74	+1·00	G2	+ 0·73	+ 1·7	− 1·2	+ 2·5	67	
83	141	7·00	+ ·01	A0	− 0·99	− 1·4	+ 1·0	− 1·8	62	
73	285	6·45	+1·52	K5	+ 0·08	− 0·9	+ 0·9	+ 0·2	72	:
78	187	6·75	+ ·28	F2	+ 0·64	− 7·8	+ 8·1	+ 0·6	67	
77	195	6·48	+ ·16	A3	+ 0·45	− 1·3	+ 1·5	+ 1·1	68	
74	241	7·19	+ ·10	A0	+ 0·25	+ 3·3	− 3·0	+ 1·5	71	
74	242	6·91	− ·03	A0	+ 0·82	− 1·6	+ 2·2	+ 3·1	71	
74	249	6·98	+ ·23	A6	+ 1·28	−10·7	+11·3	+ 3·7	71	
74	250	7·56	+ ·43	F0	− 0·21	−19·0	+18·6	− 3·6	71	
74	252	6·22	+1·54	K5	− 0·09	+ 1·9	− 1·9	− 0·1	71	
85	80	6·07	+1·58	K5	+ 1·6	+ 0·4	− 0·1	+ 2·0	61	
73	298	6·54	+ ·37	A9	+ 0·39	+ 0·1	+ 0·1	+ 1·6	72	
74	257	7·25	+ ·59	F8	+ 4·78	−19·0	+20·9	+16·9	71	
75	236	7·54	+ ·33	F2	+ 0·62	− 1·8	+ 2·0	+ 2·2	70	
75	247	6·20	+1·48	K5	+ 0·92	− 1·4	+ 1·5	+ 3·5	70	
73	311	7·29	+ ·47	F2	− 0·59	− 4·3	+ 4·2	− 2·5	72	
77	229	7·31	+ ·90	G1	+ 0·28	− 0·1	+ 0·1	+ 0·9	69	
76	226	7·65	+ ·42	F0	− 0·06	− 0·8	− 0·8	− 0·2	70	
79	198	7·88	+1·05	G±	+ 0·45	+ 0·3	− 0·3	+ 1·2	66	:
76	234	7·80	+ ·53	G±	+ 0·37	− 7·4	+ 7·4	+ 1·4	69	:
86	79	6·45	+1·24	K1	+ 3·0	− 9·7	+ 9·7	+ 3·0	59	
73	328	7·55	+1·05	G±	− 0·88	− 2·3	+ 2·5	− 3·6	73	:
78	225	7·90	+1·07	G±	− 0·75	− 3·6	+ 3·7	− 1·5	68	:
79	201	8·05	+ ·28	F0	− 0·57	− 0·9	+ 1·0	− 1·4	66	
79	202	6·59	+1·27	G8	+ 0·22	− 1·9	+ 1·8	+ 0·8	67	
80	210	6·98	+1·56	K5	+ 0·52	− 1·2	+ 1·1	+ 1·4	65	
78	226	7·03	− ·02	B9	+ 0·46	− 0·2	+ 0·1	+ 1·4	67	
76	244	7·86	+ ·58	F−G	− 1·21	− 4·1	+ 4·4	− 3·8	69	
77	247	7·49	+ ·07	A1	+ 0·86	− 2·9	+ 3·1	− 2·5	68	
79	208	6·65	+ ·09	A1	+ 0·44	+ 1·6	− 1·7	+ 1·0	66	
82	177	6·62	+ ·06	A3	+ 1·43	− 4·6	+ 4·3	+ 3·3	64	
78	227	5·50	+1·75	K5	+ 0·62	− 0·1	− 0·1	+ 1·9	68	
73	340	6·24	+ ·20	F0	− 3·54	− 5·5	+ 6·9	−14·2	72	
80	217	7·23	+1·28	K0	+ 1·22	+ 2·9	− 3·1	+ 2·8	66	:
79	212	5·39	+ ·43	F5	− 2·57	−61·6	+62·0	− 1·3	66	
76	249	7·00	+ ·96	G1	− 0·56	− 2·4	+ 2·6	− 1·6	69	
78	234	7·79	+ ·33	F1	− 0·70	− 7·9	+ 7·5	+ 3·3	68	

Catalogue of Zones +73° *to* +90°—continued.

No.	in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	υ.	τ.	λ.	
	°				s	"	"	"	°	
75	273	7·73	+ ·34	A8	− 0·24	− 0·2	+ 0·3	− 0·9	71	:
77	266	4·62	+1·38	K3	+ 2·50	− 1·4	0·0	+ 8·4	69	
75	281	6·67	+ ·55	F5	− 2·93	−27·4	+29·0	− 5·6	70	
75	280	6·88	+ ·69	G2	+ 0·23	− 0·3	+ 0·1	+ 0·9	70	
73	360	7·39	+1·02	G±	+ 1·14	− 2·1	+ 1·1	+ 5·1	72	:
87	51	4·95	+1·75	K5+	− 4·7	− 3·8	+ 4·5	− 2·5	59	
78	240	6·81	+ ·37	A6	+ 0·49	+ 0·4	− 0·7	+ 1·3	67	
73	366	7·56	+ ·43	F0	− 0·85	− 0·8	+ 1·7	− 3·3	72	
80	230	7·07	+1·22	G5	− 0·48	+ 1·5	− 0·9	− 1·5	65	
81	242	6·25	− ·03	B8	+ 0·33	− 2·8	+ 2·5	+ 1·4	64	
78	246	7·67	+ ·56	F5	− 1·00	− 0·4	+ 1·2	− 2·9	67	
75	296	6·82	+ ·03	A0	− 0·65	− 5·4	+ 5·9	− 1·0	70	
82	201	4·90	+1·72	M	+ 0·29	− 4·5	+ 4·1	+ 1·8	63	
78	250	7·11	+1·50	K2	+ 0·72	− 5·9	+ 5·1	+ 3·7	67	
78	251	7·51	+ ·07	A2	+ 0·31	− 3·4	+ 3·0	+ 1·9	67	
74	322	7·49	− ·07	A0	− 0·03	− 1·0	+ 1·0	+ 0·3	72	
75	300	...	...	A0?	+ 0·24	+ 2·1	− 2·3	+ 0·3	70	:
73	375	7·14	− ·10	A − F	+ 0·83	+ 0·6	− 1·6	+ 3·3	72	
73	376	7·51	...	A±	+ 1·07	+ 0·3	− 1·6	+ 4·3	72	
81	252	6·41	+1·21	G5	+ 0·12	− 0·3	+ 0·2	+ 0·4	65	
77	286	7·12	+1·20	G+	− 0·42	+ 0·9	− 0·4	− 1·6	68	
78	254	7·49	+ ·46	F0	− 0·64	−10·3	+10·4	+ 1·3	67	
76	284	7·60	+ ·45	F0	− 0·56	−11·7	+11·7	+ 1·8	69	
73	381	7·38	+ ·43	F+	− 0·63	− 5·9	+ 6·4	− 0·6	71	
*82	213	7·82	− ·07	A0	− 0·19	+ 0·6	− 0·5	− 0·4	64	
74	335	6·96	+1·24	G − K±	+ 1·04	− 5·3	+ 3·4	+ 5·9	71	
81	257	6·92	+1·39	K2	+ 0·87	− 3·9	+ 2·8	+ 3·2	65	
80	238	6·52	+ ·79	G1	−19·20	+ 6·2	+12·4	−46·6	65	
80	240	6·79	+1·57	K4	− 0·25	− 1·2	+ 1·3	− 0·1	66	
77	303	6·80	+ ·43	A3	+ 0·49	− 1·2	+ 0·5	+ 1·9	67	
74	338	5·48	+1·37	K2	− 0·20	− 3·3	+ 3·3	+ 0·6	71	
79	265	5·41	00	A0	− 1·15	− 6·0	+ 6·7	− 0·2	65	
*86	113	7·33	+ ·79	G0	− 3·38	− 2·9	+ 4·1	− 1·6	60	
84	169	6·36	·00	A0	− 0·4	− 2·8	+ 2·8	+ 0·7	62	
*78	278	7·73	+ ·68	F±	− 4·38	−10·1	+15·0	− 8·0	67	
*89	13	6·96	+ ·16	A3	− 3·5	+ 0·7	− 0·1	− 1·2	57	
76	308	7·50	+ ·27	F0	− 0·76	−11·1	+11·0	+ 2·7	68	
82	235	6·30	− ·05	B9	− 0·19	− 3·1	+ 2·9	+ 1·1	62	

Catalogue of Zones +73° to +90°—continued.

No. in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	ν.	τ.	$\frac{1}{\lambda}$.
76° 310	5·68	+ ·71	Go	+ 0·67	+ 1·2	− 2·2	+ 1·6	68
75 334	6·42	+ ·82	G2	+ 1·25	+ 4·2	− 6·0	+ 2·2	68
78 284	8·40	+ ·02	Ao	+ 0·84	+ 0·3	− 1·6	+ 2·1	66
77 327	6·75	+1·50	K±	+ 1·13	+ 0·9	− 2·7	+ 2·8	67
78 287	7·06	+ ·90	Go	+ 0·47	− 3·4	+ 2·2	+ 3·0	66
73 416	7·30	+1·24	Ko	− 0·05	− 1·1	+ 1·0	+ 0·4	70
73 420	7·24	+ ·93	Go	+ 1·63	+ 5·9	− 8·6	+ 2·8	70
77 337	7·11	+ ·59	F5	+ 1·26	+ 6·4	− 7·6	+ 0·5	66
75 342	6·39	+ ·04	A2	− 0·56	− 2·0	+ 2·9	− 0·8	68
85 128	7·26	+ ·36	F3	− 7·9	− 8·5	+12·5	− 3·1	60
82 253	6·73	+ ·01	Ao	− 0·90	− 0·7	+ 1·5	− 1·0	62
*74 370	6·24	+ ·83±	G5	− 0·42	−10·5	+ 9·8	+ 4·0	69
73 428	6·76	+ ·02	Ao	− 0·43	− 2·1	+ 2·8	− 0·4	70
78 291	7·70	+ ·16	F5	− 0·92	− 7·5	+ 7·8	+ 1·9	66
73 430	7·19	− ·01	Ao	− 0·05	− 0·3	+ 0·4	0·0	69
78 293	6·93	+1·61	K5 − M	− 0·04	− 0·9	+ 0·8	+ 0·4	65
74 379	7·16	+1·24	K±	− 0·22	− 1·1	+ 1·4	+ 0·1	69
80 272	7·62	− ·12	B9	− 0·72	− 0·4	+ 1·4	− 1·4	63
83 232	7·08	+1·03	G3	− 0·11	− 0·7	+ 0·6	+ 0·3	61
83 233	6·99	+ ·31	F1	+ 0·79	+ 0·2	− 1·0	+ 1·0	61
78 297	7·24	+ ·37	Fo	− 0·31	− 0·9	+ 1·3	− 0·2	65
76 335	7·74	+ ·51	F±	− 1·49	−19·2	+18·3	+ 7·8	66
79 294	7·70	+ ·13	A3	− 0·50	− 2·6	+ 2·8	+ 0·6	64
84 196	6·26	+ ·21	A6	+ 1·0	+ 1·9	− 2·4	− 0·2	60
81 282	6·27	+ ·49	F1	− 1·62	− 1·0	+ 3·1	− 1·5	62
75 365	7·35	+ ·95	G±	− 0·93	+ 3·6	− 0·5	− 5·0	67
79 300	8·21	− ·32	B4	+ 0·81	+ 1·2	− 2·4	+ 0·8	64
77 359	6·94	+ ·07	Ao	− 1·08	+ 1·7	+ 1·1	− 3·7	65 :
73 452	5·92	+ ·16	A2	− 1·75	− 6·6	+ 9·9	− 1·6	68
74 393	6·30	+1·10	G5	− 0·76	− 7·3	+ 7·5	+ 2·4	67
75 370	7·21	+ ·28	A2	− 0·45	− 0·7	+ 1·6	− 0·8	66
*79 305	8·38	...	F±	+ 1·55	+ 5·1	− 6·8	− 0·3	64 :
75 377	6·16	+ ·06	A2	− 0·36	+ 2·8	− 1·1	− 2·9	66
83 256	6·91	+ ·42	F2*p*	− 0·35	− 1·1	+ 1·2	+ 0·4	60
81 302	4·49	+1·34	K2*p*	− 0·43	− 2·5	+ 2·4	+ 1·2	62
74 402	6·43	− ·24	B9	+ 2·27	− 9·8	+ 7·5	− 7·7	66 :
*83 262	7·49	+ ·06	A3	+ 0·11	+ 4·7	− 3·3	− 3·4	61
73 470	6·29	+ ·33	A5	− 2·12	−10·4	+13·7	+ 1·0	67 :

Catalogue of Zones +73° *to* +90°—continued.

No. in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
75° 386	6·58	+ ·90	G+	+ 0ˢ·86	+ 4″·6	− 5″·5	− 1″·1	66°	:
73 471	7·05	+ ·10	A3	− 1·48	+ 3·8	+ 1·9	− 7·2	67	:
79 316	8·28	+ ·02	A2	− 0·60	− 1·5	+ 2·2	0·0	63	:
*76 360	7·22	+1·03	G+	− 0·77	− 3·7	+ 4·0	+ 1·2	65	
78 317	6·20	+1·56	K2	− 0·17	+ 0·3	+ 0·2	− 0·6	63	
75 389	6·73	+ ·13	A5	− 1·94	+ 1·4	+ 4·4	− 6·1	65	
79 319	6·14	+ ·19	A7	− 0·60	− 2·4	+ 2·9	+ 0·6	62	
78 327	7·57	+ ·93	G±	− 1·44	+ 0·4	+ 3·0	− 3·1	63	
*81 319	7·89	+ ·67	G±	0·00	+ 0·5	− 0·3	− 0·4	61	
77 389	7·74	+ ·74	G±	+ 0·04	+ 0·9	− 0·6	− 0·7	64	
73 478	5·94	+1·05	K2*p*	− 1·48	− 4·7	+ 7·9	− 0·4	66	
84 225	6·13	+1·74	K5	− 0·97	+ 1·6	− 2·1	− 0·4	59	
75 399	6·84	+ ·91	G2	− 1·49	− 3·4	+ 6·5	− 0·9	65	
*83 280	7·26	+1·05	G±	+ 1·55	− 2·7	− 0·9	+ 3·9	60	
79 328	6·85	+ ·10	A2	+ 1·03	− 0·5	− 2·0	+ 1·9	61	
73 489	6·51	+ ·25	A6	− 1·20	− 8·9	+ 9·4	+ 4·2	65	
83 287	6·73	+1·07	G±	+ 0·85	− 0·8	− 0·9	+ 1·5	59	
*79 330	7·22	+1·24	G−K±	− 0·93	− 3·6	+ 4·2	+ 1·6	61	
84 234	5·39	+ ·18	A4	− 9·0	− 4·1	+12·8	− 2·5	58	
83 297	5·07	+ ·32	A8	− 4·5	+ 2·4	+ 5·8	− 6·0	59	
*85 161	7·15	+ ·84	Go	+ 0·08	+ 5·8	− 2·8	− 5·1	58	
78 349	7·83	+1·06	G±	− 0·51	− 3·5	+ 3·1	+ 2·1	61	
*74 436	6·90	+1·03	G2	+ 1·31	− 1·8	− 3·6	+ 4·5	64	
81 343	6·19	+1·28	Ko	− 0·59	− 0·8	+ 1·6	+ 0·1	60	
74 438	7·36	+ ·13	A2	− 0·30	− 3·9	+ 3·0	+ 2·7	63	
76 393	5·11	+ ·76	G±	− 0·79	− 1·0	+ 2·9	− 0·6	62	
74 440	7·52	+ ·30	F2	− 3·24	− 8·4	+15·6	+ 0·5	63	
*81 349	6·49	+ ·22	A1	+ 0·83	+ 1·3	− 2·4	− 0·2	60	
*78 359	7·63	− ·06	A±	− 1·20	+ 1·1	+ 2·9	− 2·8	61	
76 402	7·01	+ ·26	A4	− 2·19	− 3·1	+ 8·3	− 0·7	61	
*80 338	7·59	+ ·97	G±	+ 0·46	− 0·5	− 0·7	+ 0·9	59	
77 412	6·74	+1·66	M±	− 1·11	+ 0·2	+ 3·2	− 1·7	61	
80 347	...	...	F±	+ 2·69	+ 8·2	− 9·4	− 4·8	59	
76 406	7·42	+ ·39	A8	− 4·19	− 4·9	+15·6	− 2·0	61	
78 367	6·12	+1·08	G3	− 2·49	− 2·7	+ 8·0	− 0·6	60	
79 348	7·38	+1·00	G±	+ 2·02	− 4·9	− 3·2	+ 6·6	59	
82 325	7·10	+ ·66	F8	− 5·65	−18·2	+18·3	+14·8	58	
78 375	6·92	+1·83	K±	+ 1·39	− 1·7	− 3·3	+ 3·1	60	

Catalogue of Zones +73° to +90°—continued.

No. in B.D.		Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
					s	″	″	″	°	
86°	161	7·17	+ ·19	A5	− 3·1	− 0·2	+ 3·0	− 0·6	57	
88	64	7·45	− ·18	B9	− 0·21	+ 1·2	− 0·2	− 1·2	56	
74	456	7·24	...	G−K±	− 9·47	+10·7	+32·3	−24·3	61	
74	456*a*	7·68	...		+ 0·60	− 1·9	− 1·6	+ 2·7	61	:
*79	356	6·87	+1·27	G5	− 2·10	− 1·5	+ 6·3	− 0·6	59	
*75	438	7·46	+1·09	K0	+ 1·97	− 2·5	− 6·3	+ 5·1	61	:
*78	385	7·31	+ ·53	F5	− 2·10	0·0	+ 6·3	− 2·0	59	
85	183	7·07	+ ·91	G1	− 0·33	+ 3·8	− 0·4	− 3·8	57	
81	373	6·10	+ ·14	A2	− 6·52	+ 3·9	+12·8	− 7·1	58	
86	170	7·12	+ ·36	A8	− 1·48	+ 5·7	+ 0·5	− 5·9	57	
78	392	6·34	+1·73	K5	− 0·29	+ 1·5	+ 0·5	− 1·7	58	
73	532	6·82	+1·65	K5	+ 0·09	0·0	+ 0·4	− 0·1	59	
*76	434	7·84	+ ·39	F±	− 1·10	+ 3·3	+ 4·7	+ 2·3	55	
*74	475	7·44	+ ·98	G3	− 0·06	− 0·2	+ 0·4	+ 0·2	58	:
74	476	6·65	+ ·58	F6	− 1·20	−10·0	+ 7·0	+ 8·8	58	
80	370	7·70	+ ·52	F−G±	+ 1·72	+ 0·4	+ 4·4	+ 0·2	57	
81	389	6·24	+1·64	K8	− 2·63	− 3·4	+ 6·3	+ 2·7	57	
86	176	6·15	+ ·53	F5	− 5·9	+ 8·8	+ 5·5	− 9·2	56	
77	460	7·18	+ ·46	F2	− 3·17	− 8·1	+11·6	+ 6·4	57	
77	461	5·68	+1·15	G3	+ 4·43	− 8·9	−12·9	+10·9	57	
74	484	7·24	+1·29	G±	− 1·08	+ 0·2	+ 4·4	− 1·2	58	
75	469	6·21	+ ·39	F5	+ 0·27	− 0·7	− 0·9	+ 0·8	57	
*78	406	6·74	+1·03	G1	− 4·88	+ 5·3	+14·6	− 7·1	57	
82	356	6·06	+1·23	K2	− 0·94	− 0·1	+ 1·9	0·0	56	
82	357	8·29	...	...	− 0·74	+ 0·2	+ 1·5	− 0·3	56	
*78	411	6·64	− ·10	A0	− 1·15	+ 1·1	+ 3·5	− 1·5	57	
78	412	5·20	+ ·20	A4	+ 0·32	+ 1·8	− 1·2	− 1·7	57	
84	269	8·16	+ ·28	F±	0·0	+ 1·5	− 0·1	− 1·5	56	
73	549	6·24	+1·15	G3	− 0·42	− 3·6	+ 2·3	+ 3·3	57	
*87	107	6·12	+ ·42	F0	+26·6	− 0·6	−20·7	+ 0·4	56	
75	470	5·50	− ·22	B7	− 0·69	+ 0·4	+ 2·5	− 0·7	56	
88	71	6·07	+ ·29	F0	− 7·4	+ 5·8	+ 3·6	− 5·6	56	
*84	274	7·79	+ ·47	G±	+ 0·17	+ 0·9	− 0·1	− 0·9	56	
77	475	7·22	+1·38	G±	− 1·06	− 0·2	+ 3·5	+ 0·1	55	:
75	473	7·22	+1·17	G0	+ 0·33	− 1·4	− 1·1	+ 1·4	55	:
73	559	7·28	+ ·69	F±	+ 1·86	− 3·8	− 7·9	+ 4·4	56	
*76	454	6·87	+1·45	G5±	− 2·12	− 0·6	+ 7·8	+ 0·3	55	:
*81	400	7·23	+ ·22	A6	− 3·55	− 0·7	+ 8·5	+ 1·0	55	

Catalogue of Zones +73° to +90°—continued.

No.	B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A. (s)	Proper Motion in Dec. (″)	v. (″)	τ. (″)	λ. (°)	
*80	389	7·00	+ ·50	F–G*p*	− 3·77	− 0·8	+10·0	+ 1·0	55	
86	182	7·12	+ ·16	A3	+ 1·49	+ 1·0	− 1·7	− 1·2	55	
73	561	7·08	+1·16	Go	− 0·33	+ 1·1	+ 1·4	− 1·2	55	
76	462	7·08	+1·40	K±	+ 0·17	− 2·7	− 0·6	− 2·7	55	
84	286	7·24	+ ·48	F5*p*	−14·21	+ 1·8	+21·9	0·0	55	
81	402	6·28	+ ·11	A3	+ 1·43	− 2·7	− 3·6	+ 2·4	55	
80	395	7·43	+1·37	Ko	+ 0·50	− 1·3	− 1·3	+ 1·1	55	
*84	289	5·64	...	A±	− 1·5	+ 1·4	+ 2·6	− 1·0	55	
*84	290	5·14			− 1·8	+ 1·5	+ 3·0	− 1·1	55	
*76	470	6·84	+ ·94	Go	− 0·42	− 1·1	+ 1·4	+ 1·2	54	
79	407	7·51	+1·69	K–M	− 0·22	− 1·5	+ 0·4	+ 1·5	54	
83	369	6·83	+1·28	G–K±	+ 2·1	+ 3·3	− 3·3	− 3·8	55	
76	473	6·08	+ ·97	G2	− 0·25	+ 0·9	+ 0·8	− 0·8	54	
81	412	7·15	+ ·76	Go	− 3·02	− 0·1	+ 6·7	+ 1·2	54	
86	187	7·70	+ ·37	F2	− 1·38	+ 1·4	+ 1·6	− 1·1	55	
73	583	6·52	+ ·05	A3	− 0·37	+ 1·8	+ 1·7	− 1·7	53	
73	587	6·54	+ ·11	Ao	+ 0·75	− 4·5	− 3·7	+ 4·1	53	
80	404	7·36	+1·34	Ko	+ 1·67	+ 0·1	− 4·2	− 0·9	53	
81	416	6·16	+1·05	G3	+ 0·26	+ 0·6	− 0·4	− 0·7	54	
79	418	7·45	+1·01	Go	− 2·54	+ 1·2	+ 4·8	− 5·3	53	
85	222	7·28	+ ·40	F5*p*	−10·4	+ 2·6	+13·2	+ 1·1	54	
79	422	5·75	+ ·67	G1	− 4·73	+ 2·2	+13·4	+ 1·2	53	
*84	311	7·12	+ ·70	F9	+ 2·12	+10·9	− 0·2	−11·4	54	
75	507	7·39	+1·18	G±	− 0·13	− 1·5	+ 0·2	+ 1·6	52	
76	491	7·01	+1·08	Go	+ 0·79	− 2·8	− 3·4	+ 2·1	52	
*86	193	7·96	+ ·07	F5*p*	− 2·8	+ 2·1	+ 3·6	− 0·9	54	
76	492	7·75	− ·03	Ao	− 0·08	− 4·4	− 0·7	+ 4·3	57	:
*77	515	7·81	+ ·94	G±	+ 0·03	− 0·8	− 0·3	+ 0·7	52	:
77	516	6·31	+1·79	K5	− 0·76	− 0·5	+ 2·3	+ 1·0	52	
*80	417	7·30	+1·39	K±	− 1·57	+ 2·8	+ 4·8	− 1·5	52	
77	519	6·58	+ ·09	A3	− 0·77	− 0·6	+ 2·2	+ 1·2	51	
78	466	5·79	+1·07	G8	− 2·32	+ 4·0	+ 7·8	− 1·7	52	
80	421	6·86	+1·67	K5	− 0·45	+ 1·4	+ 1·5	− 1·0	52	:
83	397	5·90	+ ·95	G3	+ 2·19	− 4·7	− 5·4	+ 2·8	53	
76	500	7·99	+ ·50	A–F	− 2·51	+ 7·5	+10·9	− 4·4	51	
76	502	7·44	+ ·23	A5	+ 0·80	− 1·1	− 3·1	+ 0·2	51	
80	422	7·12	+1·05	G5	+ 0·25	+ 0·2	− 0·5	+ 0·4	52	:
79	431	6·48	+1·24	G8	− 1·67	0·0	+ 4·3	+ 1·6	51	

Catalogue of Zones +73° to +90°—continued.

No.	in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
					s	″	″	″	°	
85°	234	7·65	+1·06	G4	− 2·52	− 1·5	+ 2·4	+ 2·7	54	
81	452	6·50	+1·30	K2	+ 2·85	− 0·1	− 6·1	− 2·3	52	
*74	560	7·60	+ ·93	G±	+ 1·01	− 1·7	− 4·5	+ 0·4	50	
*77	523	7·76	+ 1·3±	K±	+ 0·17	+ 0·3	− 0·5	− 0·5	51	
*74	563	7·61	+ ·63	F−G	+ 1·49	− 2·9	− 6·8	+ 0·8	49	
86	201	7·12	+ ·47	F2	− 1·6	+ 3·3	+ 3·0	− 2·2	54	
75	527	7·25	+1·22	G5	+ 1·80	−12·6	−11·0	+ 9·3	49	
75	529	6·29	+ ·14	A2	− 1·42	+ 0·8	+ 5·4	+ 1·3	49	
78	478	4·78	+1·59	K5	− 0·89	+ 2·6	+ 3·7	− 1·3	50	
75	532	7·57	+1·03	Go	− 0·87	− 2·8	+ 1·7	+ 3·9	48	
76	527	4·31	+1·35	K5	+ 0·35	+ 1·7	− 0·3	− 2·1	48	
*79	447	7·11	+ ·39	F3	− 0·37	+ 8·0	+ 4·9	− 6·5	50	
81	482	6·73	+ ·92	Go	− 3·73	−12·2	+ 0·5	+14·9	51	
80	448	6·11	+1·48	K3	− 2·51	+ 8·4	+10·0	− 3·4	50	
80	451	7·05	+1·68	K5±	+ 0·94	− 1·0	+ 1·4	+ 2·2	50	
76	536	7·32	+ ·16	A3	− 0·59	− 1·8	+ 0·7	+ 2·6	48	:
86	217	7·01	+1·02	G3	+ 0·21	− 0·1	− 0·2	0·0	54	
81	495	7·13	+ ·03	A1	− 1·17	− 2·5	+ 3·6	− 0·3	50	
78	501	6·45	+1·07	G5	− 0·31	− 0·2	+ 0·6	+ 0·7	48	
75	545	6·86	+1·74	K−M	− 0·39	− 0·3	+ 1·0	+ 1·1	46	
*83	431	5·49	+ ·71	F8	+ 8·97	−24·3	−28·3	+ 8·1	51	
75	547	6·76	+1·13	Go	+ 0·36	− 0·1	− 1·2	− 0·7	47	
84	335	6·72	+1·59	K2	− 1·0	+ 1·2	+ 1·9	+ 0·1	52	
74	602	7·49	+ ·19	A3	− 0·32	+ 2·5	+ 2·5	− 1·2	46	
87	143	6·74	+1·63	K5	+ 0·8	+ 1·9	+ 1·0	− 1·7	54	
74	609	6·60	+1·27	G5	− 0·49	+ 1·8	+ 2·7	− 0·1	45	
78	510	7·12	+1·50	K±	+ 1·72	+ 0·2	− 3·5	− 3·6	48	
73	672	7·34	+ ·50	F+	− 2·62	+ 8·4	+13·7	+ 1·0	44	
73	680	7·72	+1·00	...	+ 0·44	− 4·6	− 4·5	+ 2·1	44	
77	592	4·84	+1·79	K5	+ 1·37	+ 0·6	− 2·5	− 3·6	46	
80	480	6·56	...	G2	− 8·99	+11·6	+22·8	+ 8·8	49	
80	481	7·30	...	G2	− 8·40	+11·0	+21·5	+ 8·2	49	
81	517	6·86	+1·06	G2	+ 0·14	+ 3·5	+ 2·5	− 2·5	49	
76	563	7·51	+ ·11	A±	+ 0·61	− 1·7	− 2·7	− 0·3	45	:
82	463	7·01	+ ·24	Fo	+ 5·07	− 1·3	− 7·1	− 6·9	50	
85	263	6·79	+1·09	G1	− 0·38	− 6·1	− 4·5	+ 4·1	52	
*81	523	6·85	+ ·98	G5±	− 2·49	+ 5·3	+ 7·8	+ 1·4	49	
80	487	6·78	+ ·42	Fo	− 1·45	+ 3·6	+ 5·2	+ 0·6	48	

Catalogue of Zones +73° to +90°—continued.

No. in B.D.		Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	ν.	τ.	λ.	
					s	″		″	°	
78°	527	4·09	+ ·02	A1	+ 0·80	− 0·3	− 1·8	− 1·7	46	
81	531	6·79	+1·51	K5	− 0·79	− 0·4	+ 0·1	+ 1·8	49	
83	453	7·41	+ ·20	A3	+ 0·91	+ 0·4	− 0·5	− 1·5	50	
75	579	6·73	+1·09	G5	+ 0·51	− 2·0	− 2·8	− 0·3	44	
85	269	6·82	+ ·25	A5	− 4·49	+ 6·0	+ 7·8	+ 1·2	52	
76	580	7·90	+ ·77	G±	− 4·37	+ 5·2	+13·2	+ 9·6	44	
74	650	7·70	+1·25	K±	+ 0·14	− 0·4	− 0·7	− 0·2	53	:
73	707	6·81	+ ·54	F5	− 1·64	+ 6·9	+ 9·7	+ 1·4	42	
77	616	5·73	− ·14	A1	− 0·47	+ 0·7	+ 1·4	+ 0·9	45	
*81	541	7·44 *	+ ·97	G±	− 0·84	+ 5·1	+ 5·4	− 0·9	48	
76	594	5·52	− ·34	A0	+ 0·15	+ 0·7	+ 0·3	− 0·9	44	
75	586	6·16	+1·63	K5	− 1·01	+ 2·5	+ 4·1	+ 1·9	43	
73	713	6·08	− ·14	A±	− 0·40	+ 1·8	+ 2·5	+ 0·4	42	
*76	596	5·10	+ ·08	A6	− 1·82	+25·2	+25·1	− 6·7	43	
77	623	7·42	+1·55	K±	+ 0·80	− 3·9	− 4·7	− 0·4	45	
76	600	6·92	+ ·31	A3	− 0·17	+ 1·3	+ 1·4	− 0·1	44	:
76	606	7·28	+1·7±	...	− 0·48	+ 2·2	+ 2·8	+ 0·4	44	
79	498	5·61	+ ·06	A3	− 3·65	+11·4	+14·8	+ 4·2	46	
*84	361	7·03	+ ·26	A4	+ 0·39	+ 2·4	+ 2·0	− 1·5	50	
77	627	6·41	+1·02	G4	− 2·91	+27·1	+ 2·9	− 0·3	44	
*80	519	6·90	+ ·63	F−G	+ 4·56	+12·2	−15·9	− 6·1	47	
*78	562	6·98	+ ·51	F5	+ 0·64	− 2·4	− 3·0	− 0·8	45	
79	511	6·33	+1·17	K2*p*	− 0·02	+ 2·5	+ 2·3	− 0·9	46	
74	680	6·74	+ ·25	A2	− 0·41	+ 2·5	+ 3·0	+ 0·5	41	
77	634	6·13	+ ·23	F1	+ 1·76	+20·6	+17·0	−12·8	44	
75	605	7·34	+ ·53	F±	− 0·95	+ 3·7	+ 4·8	+ 2·0	42	
79	517	6·77	+1·31	K1	+ 0·50	+ 2·2	+ 1·7	− 2·0	46	
75	608	6·78	+ ·25	A5	+ 1·10	− 1·9	− 3·2	− 3·0	42	
82	498	4·46	+ ·59	G2	+ 0·72	− 0·1	− 0·5	− 1·4	48	
77	639	7·36	+ ·25	A5	− 0·15	− 0·6	− 0·4	+ 0·7	43	:
73	751	6·30	+ ·17	A5	− 0·01	− 2·7	− 2·5	+ 0·9	40	
74	695	6·98	+ ·55	F3	+ 1·17	−10·2	−11·2	− 1·0	41	
78	573	7·59	+ ·83	G±	+ 0·31	+ 2·9	+ 2·5	− 1·8	44	
77	641	6·86	− ·26	B9	− 0·52	+ 2·1	+ 1·1	+ 2·4	44	
73	754	7·23	+1·37	...	− 0·12	− 1·2	− 1·0	+ 0·9	40	
73	755	7·54	+1·4±	...	− 0·23	− 0·5	+ 0·8	+ 0·8	40	
75	612	6·84	+ ·45	Fo	+ 1·02	− 4·5	− 6·3	− 2·0	42	
81	568	6·59	+1·11	G5	+ 2·17	+ 1·5	0·0	− 5·3	47	

Catalogue of Zones +73° to +90°—continued.

No.	in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
	°				s	″	″	″	°	
75	613	6·24	+0·13	A5	+ 0·14	− 6·2	− 6·1	+ 1·3	42	
78	580	7·06	+1·34	G−K±	− 0·60	+ 1·3	+ 1·7	+ 1·4	45	
75	616	7·36	+ ·53	F±	− 2·97	− 1·3	+ 1·8	+11·3	42	
75	617	8·06	+ ·08	A±	− 0·30	+ 1·2	+ 1·4	+ 0·7	42	
77	654	7·86	+ ·84	G±	− 0·25	+ 0·3	+ 0·4	+ 0·7	44	:
74	710	7·49	+ ·27	A5	+ 0·04	+ 6·4	+ 6·3	− 1·3	41	
79	540	7·30	+ ·05	A1	− 0·06	− 3·1	− 2·9	+ 0·9	45	
80	544	5·56	+1·76	A5+	+ 1·09	+ 0·4	0·0	− 2·8	46	
76	647	7·19	+1·37	K±	− 1·13	− 4·1	− 3·3	+ 4·8	42	
83	512	7·52	+ ·59	F+	+ 4·66	+13·7	+12·4	− 9·8	49	
74	717	6·77	+1·17	K±	− 1·80	+ 3·5	+ 4·5	+ 6·7	40	
74	720	7·72	+ ·58	F−G±	− 0·08	+ 3·5	+ 3·5	− 0·1	40	
75	639	8·06	+1·00±	...	− 0·92	− 4·8	− 4·4	+ 3·8	42	
79	556	7·40	+ ·27	A8	+ 0·73	+ 2·2	+ 2·0	− 2·2	45	
73	791	7·76	+ ·07	A2	+ 0·47	− 5·7	− 5·8	− 1·5	40	:
75	640	7·82	+ ·96	...	+ 0·50	− 2·4	− 2·5	− 1·7	41	
73	796	7·75	+ ·60	F±	+ 0·58	−12·9	−13·0	− 1·6	39	
80	555	6·91	+1·68	F5+	+ 0·20	+ 0·3	+ 0·3	− 0·5	46	
77	670	7·57	+ ·41	F±	− 1·15	+ 7·5	+ 7·7	+ 3·6	43	
76	667	5·01	+ ·31	F5?	+ 1·41	+23·9	+23·8	− 5·5	43	
74	740	6·77	+ ·15	A5	− 0·27	+ 0·6	+ 0·7	+ 1·1	41	
79	564	7·52	+ ·25	A8	− 0·61	+ 6·0	+ 2·2	+ 5·8	45	
78	616	6·16	+1·56	K5	+ 0·75	+ 4·0	+ 4·0	− 2·3	44	:
76	668	7·33	+ ·29	A6	+ 0·18	+ 0·4	+ 0·4	− 0·7	42	
75	647	6·56	+ ·86	G5	+ 0·09	+ 2·4	+ 2·4	− 0·4	41	:
78	621	7·62	+ ·19	F±	+ 0·86	+ 1·0	+ 1·0	− 2·5	45	
86	269	4·41	− ·16	B9	+ 2·1	+ 4·8	+ 4·8	− 1·8	53	
75	655	6·98	− ·07	A1	+ 0·09	+ 2·9	+ 2·9	− 0·4	42	
85	294	7·59	+ ·25	A7	+ 0·81	+ 0·3	+ 0·4	− 0·8	52	
79	570	5·58	...	F±	+ 2·02	+12·2	+12·4	− 4·8	46	
79	571	5·79	...	F±	+ 1·73	+11·4	+11·5	− 4·2	46	
86	272	5·73	+ ·18	A0	+ 6·8	+ 0·6	+ 0·9	− 5·6	53	
76	678	7·66	+ ·46	F±	− 2·15	+ 1·0	+ 0·6	+ 7·7	42	
77	681	7·41	+ ·91	G±	+ 0·45	− 0·3	− 0·4	− 1·4	43	
74	757	7·84	...	...	+ 0·40	0·0	+ 0·1	− 1·6	40	:
81	622	7·18	+1·57	K±	+ 1·38	+ 3·4	+ 2·7	− 3·7	47	
82	540	7·45	− ·20	B8	+ 1·46	+ 2·0	+ 2·3	− 2·5	49	
84	412	7·27	+ ·44	F2	+ 0·57	+ 1·4	+ 1·5	− 0·6	50	

Catalogue of Zones +73° to +90°—continued.

No.	in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
					s	″	″	″	°	
86	275	7·65	+1·16	G5	+ 6·9	+ 3·8	+ 4·4	− 5·9	52	
77	696	6·98	+1·05	G5	+ 1·62	+ 4·7	+ 5·4	− 4·7	43	
79	587	6·45	+1·34	K0	− 0·15	+ 7·8	+ 7·6	+ 1·6	45	
79	590*a*	7·20	+ ·38	A7	+ 0·26	+ 1·8	+ 1·8	− 0·4	45	
77	699	5·59	+1·33	K2	− 0·08	− 0·5	− 0·6	− 0·1	44	
83	535	7·20	+1·13	G3	+ 0·23	+ 4·4	+ 4·4	+ 0·4	49	
83	536	6·17	− ·03	A0	+ 1·00	− 2·4	− 2·2	− 2·3	49	
73	831	7·36	+ ·92	G±	− 0·43	− 0·8	− 1·1	+ 1·7	39	
77	702	6·87	+ ·38	A7	+ 0·76	+ 1·7	+ 2·2	− 2·0	43	
75	678	7·46	+ ·27	A6±	+ 0·31	+ 6·5	+ 6·6	+ 0·5	41	
86	282	6·60	+1·69	M	− 0·22	+ 2·1	+ 2·0	+ 0·6	53	
73	835	5·44	+ ·77	G±	+ 0·21	+ 8·1	+ 8·0	+ 1·4	40	
75	682	5·49	− ·21	B8	− 0·34	+ 7·7	+ 7·1	+ 3·3	42	
74	792	7·18	+ ·76	G±	+ 8·34	+ 3·4	+12·6	−32·8	41	
79	604	6·39	+ ·32	A4	+ 1·34	+ 8·7	+ 9·3	− 1·1	46	
78	660	7·99	+ ·54	F±	+ 0·38	+ 7·8	+ 7·8	+ 1·1	45	
78	661	7·84	+ ·38	A6	+ 0·26	+ 0·7	+ 0·8	− 0·5	45	:
75	683	6·39	− ·15	A0	+ 0·47	+ 0·6	+ 1·1	− 1·6	42	
73	845	7·09	+ ·22	A5	+ 0·34	− 5·9	− 5·0	− 3·4	41	
76	712	6·65	+ ·38	A9	− 1·17	− 6·9	− 7·8	+ 1·5	43	
83	547	6·83	+ ·09	A2	+ 0·43	− 2·5	− 1·4	− 2·1	50	
82	572	6·88	− ·03	B9	+ 1·34	+ 4·7	+ 5·3	− 1·1	49	
80	604	7·90	+1·07	G±	+ 1·94	+ 2·4	+ 3·9	− 3·9	47	:
77	715	7·54	+1·59	K±	+ 0·71	− 4·7	− 3·6	− 3·7	44	
79	614	7·96	+ ·84	F−G±	− 1·43	−10·8	−11·5	− 0·2	46	
76	717	5·43	− ·14	A5	+ 1·32	−12·7	−10·1	− 9·1	43	
80	607	7·28	+1·07	G0	+ 0·02	+ 0·7	+ 0·7	+ 0·2	47	
80	609	7·57	+ ·20	A2	+ 1·04	+ 1·43	+ 2·3	− 1·9	47	
73	857	4·92	+ ·92	K0	− 3·18	+11·0	+ 4·3	+17·2	40	
73	860	7·04	+ ·05	A5	+ 0·35	+ 5·6	+ 5·8	+ 1·1	40	
88	112	6·23	+1·86	M	− 1·03	+ 1·1	+ 0·1	+ 3·0	56	
76	732	7·30	+1·25	G5	+ 0·17	− 6·2	− 5·4	+ 3·1	43	
76	734	6·34	+2·72	N	+ 0·60	− 0·9	− 0·1	− 2·3	44	
79	628	6·17	− ·12	A2	+ 0·91	− 3·7	− 1·6	− 4·2	46	
83	552	6·43	+ ·17	A2	+ 1·54	+ 3·5	+ 4·3	− 1·1	50	
73	863	7·68	+ ·19	A5±	+ 0·28	+ 2·7	+ 2·9	+ 0·2	41	
81	666	7·62	+1·40	K±	+ 0·59	+ 1·7	+ 2·0	− 0·5	48	

Catalogue of Zones +73° to +90°—continued.

No.	in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	ν.	τ.	λ.	
	°				s	"	"	"	°	
*77	734	7·36 / 7·99	...	A6±	+ 2·16	− 2·2	+ 1·2	− 6·9	45	:
74	831	6·96	+1·15	G5	+ 0·48	+ 2·4	+ 3·0	− 0·5	42	
79	638	8·46	+ ·29	F±	+ 0·38	− 1·8	− 1·1	+ 1·8	47	
79	645	7·96	− ·19	A0	+ 0·75	− 1·0	+ 0·2	− 2·3	47	
79	648	7·32	+1·06	G+	− 0·95	− 2·2	− 3·3	+ 1·1	47	
78	694	7·62	+1·5±	...	+ 1·18	+ 1·0	+ 2·8	− 2·4	46	:
75	718	7·22	+1·50	K5±	+ 0·76	− 2·5	− 0·3	− 3·8	44	:
76	771	6·17	+1·78	K5	− 0·61	− 5·5	− 5·7	− 1·6	45	
73	897	6·82	+ ·95	G2	+ 0·72	+ 2·9	+ 4·2	− 0·5	43	
79	660	6·76	+ ·87	G0	+ 0·40	+ 0·4	+ 1·0	− 0·6	47	
77	764	4·63	− ·41	B5	+ 0·38	+ 2·6	+ 2·8	+ 0·7	46	
85	340	7·72	+2·0±	K+	− 0·75	+ 0·8	+ 0·1	+ 1·2	52	
84	451	6·79	+ ·17	A2	− 0·8	− 4·2	− 4·1	− 1·5	51	
81	699	7·35	+ ·48	F*p*	+ 1·14	+ 4·8	+ 5·3	− 1·0	49	
79	668	6·81	+1·43	K±	− 0·58	− 3·4	− 3·6	− 1·0	47	
80	650	6·80	− ·11	B9	− 0·87	− 1·2	+ 0·5	− 2·4	48	
84	462	7·19	+ ·28	A8	+ 3·33	+ 4·5	+ 6·6	− 1·0	51	
84	463	7·03	+ ·59	F5	− 5·90	− 0·5	− 5·6	+ 6·0	52	
75	739	7·68	+1·47	G−K ±	+ 0·26	− 0·9	0·0	− 1·3	45	
*81	706	6·89	+ ·54	F2	+ 0·29	− 4·3	− 2·2	+ 3·2	50	
79	675	7·54	− ·14	A0	+ 0·51	− 0·3	+ 0·6	− 1·1	48	
74	872	5·12	+ ·12	A±	+ 0·27	− 1·2	0·0	− 1·6	45	
*80	657	6·49	+1·10	G4	− 1·78	+ 0·5	− 2·4	+ 3·3	49	
82	617	6·67	+1·05	G2	+ 0·10	+ 0·4	+ 0·4	+ 0·1	51	
*80	659	5·48	+1·05	G4	+ 1·22	+ 1·3	+ 3·0	− 1·2	49	
80	660	5·97	+1·00	G2	+ 2·96	+21·6	+20·5	+10·0	49	
75	752	7·34	+1·01	G+	+ 9·94	+52·3	+63·3	+13·4	45	
83	588	6·21	+ ·16	A2	+ 1·28	− 2·6	− 0·4	− 3·5	51	
*78	716	6·97	− ·32	B2	− 0·10	+ 0·9	+ 0·4	+ 0·8	48	
76	809	7·03	+ ·35	F±	+ 0·74	+ 5·0	+ 5·3	+ 1·9	46	
81	712	6·92	+1·19	K±	+ 2·80	+ 4·3	+ 7·4	− 1·3	50	
*81	718	5·81	− ·03	B8	+ 1·63	+ 2·6	+ 4·2	− 0·4	50	
80	672	5·49	+1·18	G8	− 1·01	− 3·0	− 4·0	− 0·6	49	
74	890	7·17	+ ·12	A2±	+ 0·51	− 1·6	+ 0·7	− 2·5	45	
75	764	5·91	+1·07	G2	+ 1·53	+ 3·4	+ 6·6	− 0·7	46	
75	765	6·62	+1·41	K5	+ 2·17	+ 2·6	+ 8·0	− 2·8	46	
73	922	8·01	>+ ·9±	G±	+ 0·25	+ 3·4	+ 2·7	+ 2·2	46	

Catalogue of Zones +73° *to* +90°—continued.

No. in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.
				s	″	″	″	°
77° 800	6·03	− ·20	A0	+ 0·83	+ 3·3	+ 4·0	+ 1·2	48
80 679	6·92	+ ·25	A6	− 0·26	+ 1·7	+ 0·9	+ 1·6	50
74 907	6·80	+ ·64	F5	+ 0·42	− 0·5	+ 1·2	− 1·4	47
80 682	6·99	+1·36	G−K±	− 0·20	− 0·5	− 0·7	− 0·1	50
78 742	7·46	− ·02	A0	+ 0·37	− 1·4	+ 0·1	− 1·8	49
75 778	7·00	− ·15	B8	+ 0·64	+ 4·0	+ 4·1	+ 2·1	47
78 744	7·17	− ·13	B5	+ 0·70	+ 0·8	+ 2·1	− 0·5	49
80 688	7·36	+ ·01	A1	+ 1·38	+ 1·1	+ 3·5	− 1·1	50
76 833	5·92	+1·71	K5−M	+ 0·49	+ 1·0	+ 2·0	− 0·1	48
80 690	6·05	+ ·16	A2	0·00	− 0·8	− 0·5	− 0·6	50
*77 811	7·43	− ·30	A0	− 0·33	− 0·1	− 0·9	+ 0·5	49
86 319	7·43	− ·02	A2	+ 2·2	+ 1·3	+ 2·8	− 0·5	54
83 603	6·80	+1·30	K±	+ 1·3	− 1·1	+ 1·0	− 2·2	52
*81 737	7·80	+ ·44	F±	+ 1·93	+ 9·2	+ 8·6	+ 5·1	51
76 836	6·55	+ ·07	A0	− 0·03	− 2·7	− 1·4	− 2·3	48
*75 787	6·79	+ ·64	F5	− 2·73	+14·1	− 1·5	+17·1	48
75 788	7·19	+1·16	G5	+ 1·58	− 0·2	+ 5·5	− 2·1	48
*79 707	6·01	+ ·78	G1	+ 1·79	− 0·9	+ 3·4	− 3·1	50
75 791	7·66	+ ·31	A7	+ 0·48	− 2·8	+ 0·3	− 3·4	48
77 823	7·19	+1·14	G5	+ 0·24	− 0·2	+ 0·6	− 0·5	49
*72 991	8·03	+ ·51	A−F±	+ 0·95	+ 0·5	+ 4·0	− 1·4	47
74 926	7·92	+ ·21	A2	+ 0·47	+ 1·6	+ 2·4	+ 0·7	48
77 834	7·02	+ ·00	A0	+ 0·14	− 0·2	+ 0·3	− 0·3	50
83 618	7 08	+ ·27	A4	+ 5·66	+ 1·3	+ 9·0	− 3·3	53
77 836	7·77	+ ·9±	G±	− 0·21	+ 0·2	− 0·5	+ 0·5	50
*72 1003	6·73	− ·05	A0	+ 1·07	+ 2·9	+ 5·3	+ 1·2	49
78 768	6·46	+1·49	K5	+ 1·04	+ 2·6	+ 2·2	+ 1·9	51
79 721	6·39	+1·49	M	+ 0·69	− 0·3	+ 1·5	− 1·0	51
74 946	6·47	+1·58	K5	+ 0·17	− 0·5	+ 0·6	− 0·7	49
75 808	7·75	+ ·91	G±	+ 0·33	+ 1·3	+ 1·7	+ 0·9	50
82 673	7·02	...	F5	− 6·91	− 3·3	−14·0	+ 2·5	52
82 674	7·39	...	F5	− 7·03	− 2·6	−13·8	+ 3·4	52
75 818	6·88	+1·46	G−K±	+ 1·02	+ 1·5	+ 4·0	+ 0·3	51
75 820	6·62	− ·12	B7	+ 0·68	+ 0·8	+ 2·7	+ 0·1	51
75 822	7·42	+ ·02	A1	+ 0·71	+ 1·3	+ 2·9	+ 0·6	51
*83 630	7·46	+ ·04	A1	− 0·90	− 1·3	− 1·8	− 0·7	54
85 383	5·24	− ·18	B8	+ 5·1	+ 4·9	+ 7·2	+ 2·6	54
85 384	6·49	+1·38	K0	+ 3·8	+ 1·5	+ 4·6	− 0·1	54

Catalogue of Zones +73° to +90°—continued.

No. in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	ν.	τ.	λ.	
				s	″	″	″	°	
77° 860	6·68	− ·03	B8	− 0·12	+ 2·9	+ 0·3	+ 2·9	52	:
81 775	6·90	+ ·53	F3	+ 3·07	+ 7·0	+ 8·7	+ 4·4	53	
76 859	7·36	+ ·13	A5	+ 0·17	− 2·6	0·0	− 2·7	52	
87 205	7·43	+ ·05	A2	+ 5·4	+ 0·8	+ 4·1	− 0·8	55	
78 796	5·63	+ ·39	A2	− 0·28	− 3·7	− 1·8	− 3·4	52	
*79 739	6·56	+1·19	G6	+ 1·82	+ 3·2	+ 5·5	+ 1·8	53	
84 509	7·16	+ ·92	G2	+ 2·21	− 0·4	+ 2·8	− 1·4	54	
78 801	5·31	+ ·27	A2	+ 0·15	− 1·3	+ 0·1	− 1·4	52	
75 836	5·71	− ·13	B8	− 0·43	− 0·1	− 1·1	+ 0·1	52	
73 982	6·80	+1·56	K5	+ 0·25	+ 1·9	+ 1·3	− 1·6	51	:
*72 1049	5·15	+ ·28	F5	+ 3·91	+ 2·1	+17·0	− 0·9	51	
74 978	5·72	+1·83	K5+	+ 1·43	− 0·9	+ 5·4	− 1·9	52	
80 731	6·72	+ ·78	F5	+ 0·99	+ 0·5	+ 2·4	0·0	53	
*76 870	7·22	+ ·24	A4	− 0·25	+ 2·3	− 0·4	+ 2·4	52	:
73 989	7·54	+ ·73	F5±	+ 1·96	− 4·2	+ 7·8	− 5·4	52	
73 991	7·54	>1·2±	G5±	+ 0·51	+ 4·7	+ 2·8	+ 4·4	52	
77 871	7·31	− ·14	B8	+ 1·10	− 1·2	+ 3·1	− 1·8	53	
*73 994	7·20	+1·33	K±	− 0·03	− 2·0	− 0·3	− 2·0	52	
79 750	7·06	+ ·82	G0	+ 1·33	+ 1·9	+ 3·8	+ 1·2	53	
82 703	4·62	+1·50	K2	+ 1·76	+ 5·4	+ 4·5	+ 4·5	54	
79 756	6·98	+ ·95	G1	+ 0·86	− 0·9	+ 2·1	− 1·4	53	
84 513	7·01	+1·10	G5	+ 6·5	− 3·5	+ 8·7	− 5·7	55	
*73 998	7·28	+ ·58	F−G	− 1·09	− 2·5	− 4·7	− 2·1	53	:
84 517	5·74	+1·51	K4	+ 1·40	+ 9·6	+ 4·0	+ 8·9	55	
77 879	7·53	+ ·72	G±	− 3·97	+ 3·8	−11·8	+ 5·5	53	
83 640	4·57	+1·62	K6	+ 6·3	+ 3·0	+10·7	+ 0·7	55	
79 759	7·22	+1·03	G5	+ 1·67	+ 4·7	+ 5·0	+ 3·9	54	
75 867	7·32	+ ·96	F−G±	+ 1·11	0·0	+ 4·1	− 0·3	53	
76 892	7·70	+ ·24	A+	− 0·06	− 0·7	− 0·3	− 0·6	53	:
*79 761	6·69	+ ·22	A5	+ 2·11	+ 2·8	+ 5·8	+ 2·0	54	
*74 1002	6·62	+ ·02	A0	+ 0·75	+ 0·4	+ 2·9	+ 0·3	54	
74 1006	4·36	+ ·75	F8	+ 0·30	− 2·5	+ 1·1	− 2·6	54	
*79 769	7·22	+ ·12	A3	+ 0·22	− 2·5	+ 0·3	− 2·6	54	
73 1023	5·78	− ·03	A0	+ 1·23	+ 0·5	+ 5·2	− 0·5	54	
78 825	8·04	− ·31	B8±	+ 0·07	− 0·1	+ 0·2	− 0·1	54	
77 896	7·35	+ ·26	A4	− 0·31	− 0·6	− 1·0	− 0·6	54	
74 1016	6·19	+ ·13	A0	+ 0·63	+ 0·2	+ 2·5	+ 0·1	54	
72 1096	7·05	+1·30	G−K±	− 1·36	− 2·6	− 5·8	− 2·6	54	

Catalogue of Zones +73° to +90°—continued.

No. in B.D.	Visual Mag.	Colour-Index.	Spectral Type.	Proper Motion in R.A.	Proper Motion in Dec.	v.	τ.	λ.	
				s	″	″	″	°	
73° 1035	7·10	+ ·39	F1	+ 1·40	+ 2·8	+ 6·2	+ 2·0	55	
*78 835	7·57	+ ·18	A2	+ 1·16	+ 1·7	+ 5·2	+ 1·6	55	
85 399	6·50	+ ·29	A6	+ 1·39	+ 2·9	+ 1·8	+ 2·8	56	
74 1022	6·59	− ·03	A0	− 0·37	− 1·5	− 1·4	− 1·5	55	
77 908	7·10	− ·06	A0	+ 0·02	+ 0·9	0·0	+ 0·9	55	
86 344	5·54	+ ·08	A4	+ 9·5	− 1·8	+ 7·8	− 2·5	56	
77 909	7·03	+ ·03	A0	+ 1·36	+ 1·4	+ 4·5	+ 1·5	55	
82 728	7·35	+1·09	K±	+ 0·95	0·0	+ 1·9	0·0	55	
74 1032	5·79	+ ·16	A3	+ 0·67	+ 0·5	+ 2·5	+ 0·7	56	
73 1047	5·91	+1·03	G2	+ 0·04	+ 1·0	+ 0·1	+ 1·0	56	
*76 928	3·56	+ ·72	G0	− 1·79	+15·8	− 6·9	+15·4	56	
*76 934	6·56	+ ·33	F1	+ 7·86	− 8·7	+27·1	− 5·8	56	
74 1047	6·33	+ ·96	G2	+ 8·45	+ 5·2	+31·9	+ 9·7	56	
73 1063	6·47	+ ·10	A0	− 0·28	− 0·7	− 1·1	− 0·9	57	
74 1051	7·62	+1·01±	G±	− 0·02	0·0	− 0·1	0·0	57	:
*73 1068	7·80	+ ·30	A8	+ 0·64	− 0·3	+ 2·6	+ 0·1	57	
*72 1135	6·40	+ ·26	A±	+ 1·40	− 1·0	+ 6·2	+ 0·2	57	
82 748	7·16	+ ·11	A0	− 0·66	0·0	− 1·3	− 0·1	56	
79 799	7·82	+ ·09	F0	+ 1·57	+ 2·1	+ 2·8	+ 2·4	56	

Although the Harvard classification represents well the gradual progression in the spectral type of stars, yet the separation into classes having a distinct designation must be regarded as more or less arbitrary, and indeed the mean proper motions are found to vary considerably between the various subdivisions of any given type. In the following discussion the stars have been grouped into five classes, in such a way as to bring into evidence as clearly as possible the differences between the various groups. This procedure was adopted by Boss, but the present grouping was made quite independently of Boss's, and in fact is somewhat different from that which he decided upon. The resultant proper motions were plotted against the spectral type, and the division into groups was made at the points where the differences between the two adjacent groups were most clearly marked.

The five groups adopted have the following limits:

(1) Bo–B7; (2) B8–A4; (3) A5–F9; (4) Go–G5; (5) G6–M. The first three groups have the same limits as the corresponding groups of Boss, except that O stars are included in his first group, whereas none of this type occur in the stars here discussed. Boss then considers the entire groups G, K, M separately.

The stars in the first group, although only 11 in number, all have very small proper motions, less than 3″·5 per century, with

an average of 1″·93. There is a marked increase for the subdivisions B8, B9, whose average is 4″·00, and these have accordingly been included in the second group along with A0–A4 stars, with which they seem more allied than with the remaining B stars. The same distinction was found by Boss, whose average proper motions for the groups Oe5–B7 and B8, B9 were respectively 2″·40 and 3″·82. In the third group there is a rapid increase in the number of large proper motions, and a corresponding decrease in the number of small ones. In the fourth group, whilst there are a considerable number of big proper motions, there is a large relative increase in the number of small ones. The fourth group contains no proper motions larger than 18″ per century. The numbers of stars in each class and their mean total proper motions are given in Table I. In taking the means, two very large proper motions of over 50″ per century have been excluded, as has also one star of spectral type N.

TABLE I.

Group No.	Limits.	No. of Stars.	Mean Centennial P. M.
			″
1	B0–B7	11	1·93
2	B8–A4	188	3·67
3	A5–F9	187	8·25
4	G0–G5	141	6·23
5	G6–M	140	3·92

In the above catalogue it will be seen that there are a number of stars whose exact spectral types are to some extent uncertain or doubtful. In order to have as much material as possible to work with, each of these has been assigned to one or other of the above classes according to the value of its colour-index. Parkhurst has shown that there is a regular gradation between the mean colour-indices of the various types, as is shown in Table II.

TABLE II.

Type.	Mean Colour-Index.	Type.	Mean Colour-Index.
	m		m
B5	−0·28	G0	+0·86
A0	−0·06	G5	+1·08
A5	+0·18	K0	+1·32
F0	+0·40	K5	+1·55
F5	+0·63	M	+1·77

Thus if spectral type is plotted against the mean colour-index, the plotted points lie upon a straight line; and consequently if the colour-index of a star is known, its spectral type can be assigned

from the above table, with small probable error. The only uncertainty introduced by so doing is in the case of stars which are near the border-line between two of the above groups; but on the average this can cause but little error.

The v or apical component of the proper motion of a star consists of two parts, one of which is parallactic and arises from the motion of the solar system, whilst the other is due to the motion of the star itself. If M is the apparent parallactic motion and Δ is the angular distance of the star from the solar apex, the value of the former component is $M \sin \Delta$. The second components due to the peculiar motions have a random distribution, and by taking the algebraic mean of the apical motions of a large number of stars these will be eliminated, and the mean parallactic motion components for the group of stars considered will be obtained. Dividing this by the mean value of $\sin \Delta$ will give the mean parallactic motion of the group. From these results the mean parallaxes of the groups of stars can be obtained: assuming that the solar velocity is 19·5 kms. per second, and that the radius of the Earth's orbit is 149,500,000 kms., it easily follows that

$$M'' = 410\pi'',$$

M″ being in seconds of arc per century.

In this way, the results in Table III. have been obtained, and the corresponding results found by Campbell, based upon radial velocity determinations, and by Boss, in the discussion of the P.M.'s in the P.G.C., are given for comparison.

TABLE III.

Jones.			Boss.*		Campbell.†	
Classes.	M.	Mean π''.	Classes.	Mean π''.	Classes.	Mean π''.
Bo–B7	1″·28	″·0031	Oe5–B7	″·0066	Bo–B5	″·0060
B8–A4	2·38	·0058	B8–A4	·0100	B8, B9	·0129
A5–F9	4·54	·0110	A5–F9	·0121	A	·0166
Go–G5	3·13	·0076	G	·0076	F	·0354
G6–M	2·31	·0056	K	·0098	G	·0223
			M	·0080	K	·0146
					M	·0106

As the present determination of the mean parallax of the first group depends upon only 11 stars, the value obtained must be regarded as merely indicating the order of magnitude of that quantity. The remaining values are probably fairly accurate. On comparing the three sets of values of the mean parallaxes, it is at

* Calculated from Boss's values of M, *Astron. Journ.*, Nos. 623–4, p. 198.
† *Lick Obs. Bulletin*, vol. vi., No. 196, p. 134.

once evident that the present ones are considerably smaller than those obtained by Boss and Campbell. It must be remembered, however, that those values were derived from stars considerably brighter in the mean than the ones contained in Parkhurst's Catalogue, the mean magnitudes of which for the spectral classes B, A, F, G, and K to M are respectively 6·72, 6·76, 6·99, 6·86, 6·59, with a resultant mean of 6·80, whereas Campbell's mean magnitude was 4·28, and Boss's was a little fainter than 5·0. As far as the ratios of the mean parallaxes of the various groups are concerned, the present values are in good agreement with Campbell's: bearing in mind the fact that the two groupings are somewhat different, it will be seen that for each group the mean parallax is about one-third of that found by Campbell for the corresponding group. This diminution in brightness corresponds to a difference of $2^{m}\cdot5$, or to a ratio of 1 : 10 in apparent luminosity. The results may be compared with Kapteyn's tables of mean parallaxes for Type I. and Type II. stars. Assuming Type I. to end at A5 and Type II. at K5, the values obtained for the mean parallaxes of the two types are $''\cdot0056$ and $''\cdot0088$, as compared with Kapteyn's values for $6^{m}\cdot8$ of $''\cdot0058$ and $''\cdot0136$; moreover, if the value 4·1 be used as above for the solar motion—instead of 3·5, upon which Kapteyn's results are based—these latter become $''\cdot0050$ and $''\cdot0115$. The value for the Type II. stars is less than Kapteyn's value, but the comparison is made somewhat difficult on account of the uncertainty attaching to the exact limits of the two types. For a decrease in magnitude of 2·5, Kapteyn's tables give a diminution in the parallax in the ratio of 1 : 2·3, as compared with the ratio 1 : 3, obtained by comparison of the results above with Campbell's. There is not so great a difference between the present values and Boss's, which are much less than Campbell's. This is due to the fact that Boss excluded from his discussion all stars with a proper motion of over 20″ per century, and this has reduced the mean parallaxes, particularly in the case of stars of types F and G, where the number of big proper motions is relatively large.

From the figures it appears that stars of type B7 or earlier are on the average the most distant from the Sun, and that the nearest stars are in the group A5–F9, which are on the average at only two-thirds of the distance of the G0–G5 stars. These in their turn are about at three-quarters of the distance of the G6–M stars, which are approximately equidistant to the B8–A4 group.

Having obtained the mean values for the separate groups of the components of the apical motions which are parallactic in origin, these were taken out and the average arithmetical value of the apical peculiar motions was obtained. These are given below in Table IV. It will be seen that they exhibit on the whole the same features as were shown by the total proper motions. In the same table are given the arithmetic mean values of the τ or cross components. Before proceeding with the discussion, it was necessary to correct these for the effect of the probable errors. For stars with

large cross motions the effect of errors of determination will be such as to make some of the proper motions too large and others too small, and on the average the two classes will be equal. For the very small cross proper motions, however, the case is different, for whichever way the error acts it will increase the value. Thus too few small cross motions will be obtained, and the average will be too large. To correct for this, a value had to be assigned for the probable error of a single determination of a cross proper motion. Considering the number of Boss and Carrington stars in the catalogue, and the large number of other well-determined proper motions, it seemed that the probable error could not exceed $0''{\cdot}75$ per century, and this was the value adopted. There were not sufficient stars in the first group to treat in this way, but the remaining four groups have been corrected for an error of this amount. This was done by using the formula derived for this purpose by Professor Eddington.*

If $u(\tau)d\tau$ is the number of stars obtained with cross proper motions between τ and $\tau+d\tau$, and if $v(\tau)d\tau$ is the true number, then for a small accidental error the correction which must be applied to $u(\tau)$to obtain $v(\tau)$ is given by

$$-\left(\frac{1{\cdot}046\times\text{probable error}}{\text{tabular interval}}\right)^2\times\text{tabular second difference of } u(\tau).$$

It was, however, found more convenient, instead of working with the function $u(\tau)$, to use the function $\int_0^\tau u(\tau)d\tau$, which gives the number of stars with cross motions between the limits 0 and τ. The values obtained for this function for $\tau=''{\cdot}25$, $''{\cdot}5$, $''{\cdot}75$, $1''{\cdot}0$, etc., were plotted, and a smooth curve drawn through them, from which the first differences were read off. These were again plotted against τ and another smooth curve drawn, from which the second differences were read off for values of τ at intervals of $''{\cdot}25$. The correction calculated from the above formula was then applied to the values initially obtained, and the resulting corrected mean values of the cross proper motions were determined. On account of the uncertainty as to the exact value of the probable error of a proper motion determination, the values finally obtained are necessarily somewhat uncertain, but they have been undoubtedly improved by the correction.

In Table IV. are given in column 1 the group limits, in column 2 the mean apical peculiar components of proper motion, in column 3 the original mean value of the cross motion, and in column 4 the mean value obtained after correcting for a probable error of $0''{\cdot}75$ per century.

* *M.N., R.A.S.*, vol. lxxiii., 1913, p. 359.

TABLE IV.

Group Limits.	Mean Apical Peculiar P.M.	Mean Cross Motion. Uncorrected.	Mean Cross Motion. Corrected.
	″	″	″
Bo–B7	1·51	0·55	(0·55)
B8–A4	2·55	1·64	1·41
A5–F9	6·45	3·29	3·09
Go–G5	4·68	2·88	2·56
G6–M	2·95	2·06	1·88

The ratio of the mean apical and cross components of the proper motions is related to the ratio of the diameters of Schwarzschild's ellipse, in his theory of star-streaming. The exact interpretation of this ratio on that theory is not evident; but by making certain assumptions, some light can be thrown upon this point. Working on the supposition that stellar velocities are distributed according to the Maxwellian law for the molecules of a gas—a law which is supported by observation,—Dr. Dyson * has shown that a distribution of the Carrington stars in space such that $N 2k^2 r e^{-k^2 r^2} dr$ is the number between distances r and $r+dr$, is in good agreement with the numbers of cross proper motions between various limits. Professor Eddington † also found the same law to fit best the observations for the bright stars. This law of spatial distribution will accordingly be assumed for the present: how far the assumption is justifiable is discussed later. The theory for the cross motions has been worked out by Dr. Dyson in a second paper.‡ When there is a stream velocity V, the number of stars between the distances r and $r+dr$ is

$$N \cdot 2k^2 r e^{-k^2 r^2} dr \cdot \frac{h}{\sqrt{\pi}} e^{-h^2(v-V)^2} dv.$$

If v be the component of proper motion corresponding to a component of velocity $\frac{1}{r}v$, then $rv = v$, and the algebraic mean value of the cross motions is

$$= 2k^2 \int_0^\infty e^{-k^2 r^2} dr \int_{-\infty}^{\infty} \frac{h}{\sqrt{\pi}} v e^{-h^2(v-V)^2} dv$$
$$= k\sqrt{\pi} V.$$

The mean apical proper motion is the arithmetic mean of the residuals from this value. This cannot be obtained exactly, but since hV is shown subsequently to be comparatively small, an approximation can be obtained by neglecting powers of hV higher than the third.

* *M.N.*, *R.A.S.*, vol. lxxiii., 1913, p. 334.
† *M.N.*, *R.A.S.*, vol. lxxiii., 1913, p. 350.
‡ *M.N.*, *R.A.S.*, vol. lxxiii., 1913, p. 402.

The number of stars with proper motions between v and $v+dv$ is

$$N\frac{h}{\sqrt{\pi}}\frac{dv}{\sqrt{\pi}}\cdot\int_0^\infty 2k^2r^2\mathrm{Exp}\{-(k^2+h^2v^2)r^2+2h^2v\mathrm{V}r-h^2\mathrm{V}^2\}dr,$$

or, on expansion as far as $h^3\mathrm{V}^3$, and integration with respect to r,

$$= N\cos\theta d\theta\left[\frac{1-h^2\mathrm{V}^2}{2}+\frac{2h\mathrm{V}}{\sqrt{\pi}}\sin\theta+\frac{3}{2}h^2\mathrm{V}^2\sin^2\theta+\frac{4h^3}{\sqrt{\pi}}\mathrm{V}^3\left(\frac{2}{3}\sin^3\theta-\sin\theta\right)\right]$$
$$= N\cos\theta\phi(\theta)d\theta \qquad \text{(say)}$$

where $$hv=k\tan\theta.$$

For the mean value of v, $\theta=\theta_1$ given by $\tan\theta_1=h\sqrt{\pi}\mathrm{V}$.

The arithmetic mean of the residuals is accordingly

$$\frac{k}{h}\int_{\theta_1}^{\frac{\pi}{2}}\cos\theta(\tan\theta-\tan\theta_1)\phi(\theta)d\theta+\frac{k}{h}\int_{-\frac{\pi}{2}}^{\theta_1}\cos\theta(\tan\theta_1-\tan\theta)\phi(\theta)d\theta$$
$$=\frac{k}{h}\left(1+h^2\mathrm{V}^2\frac{\pi-2}{2}\right)$$

as far as terms in $h^3\mathrm{V}^3$, upon evaluation.

Since the values of this expression and of the mean $k\sqrt{\pi}\mathrm{V}$ are known, $\frac{k}{h}$ can be derived by a process of successive approximation: and the ratio of this value to the value for the cross motions (which, as is seen by putting $\mathrm{V}=0$, is simply the arithmethic mean cross motion) gives the ratio of the axes of Schwarzschild's ellipse. The term $h^2\mathrm{V}^2(\pi-2)/2$ in the cases here considered is less than 0·1, which justifies the approximation.

In Table V. are given the apical proper motions of the various groups, the deduced values of k/h, the corresponding values for the cross motions, and their ratio; also in the last column the deduced value of $h\mathrm{V}$. Since the apical P.M.'s have not been corrected for the effect of a probable error, the uncorrected value is also used for the cross P.M.'s.

TABLE V.

Group.	Mean Apical P.M.	$\frac{k}{h}$ (apical).	$\frac{k}{h}$ (cross).	Ratio.	$h\mathrm{V}$.
	″	″	″		
B8–A4	2·55	2·18	1·64	1·33	·505
A5–F9	6·45	5·93	3·29	1.80	·354
Go–G5	4·68	4·35	2·88	1·51	·332
G6–M	2·95	2·66	2·06	1·29	·402

By another method Dr. Dyson obtained for the Carrington stars a ratio of 1·81 and a value of $h\mathrm{V}=·311$.

The law of spatial distribution which has been assumed is shown below to be in good agreement with the proper motion data

for the second and third of the above groups, but not to agree very well for the first and fourth, so that for these two groups the ratio of the axes of the Schwarzschild ellipse here obtained may not be very accurate, although this value is a better approximation than would be furnished by the ratio of the mean apical and cross proper motions.

From the values of the ratio of the two axes thus deduced it appears that star-streaming is strongly marked in the group A5–F9, and is shown in a lesser degree in the group G0–G5. The remaining two groups B8–A4, and G6–M show but a slight tendency towards streaming.

Since the mean parallactic motion obtained above is equivalent to the solar motion, the values of the mean cross motion can be at once turned into velocities, with the results given in Table VI., where the corresponding Lick Observatory radial determinations are given for comparison, as also Boss's values for the mean cross motion. The value of the solar velocity has been assumed to be 19·5 kms. per second.

TABLE VI.

Group Limits.	Mean Cross P.M.	M.	$\frac{\tau''}{M}$.	τ in kms. per sec.	L.O. Mean Radial Velocity.	Boss.
	″	″				
B0–B7	0·55	1·28	·430	8·4	6·2	6·3
B8–A4	1·41	2·38	·592	11·5	9·8	10·2
A5–F9	3·09	4·54	·681	13·3	13·5	16·2
G0–G5	2·56	3·13	·818	16·0	16·0	18·6 *
G6–M	1·88	2·31	·814	15·9	17·0	15·1 * 17·1 *

If, as in Boss's discussion, the stars considered were distributed uniformly over the whole sky, the mean values of the cross motion, when expressed as a velocity, should be equal to the mean radial velocity. Here, however, the comparison is complicated by the fact that the mean cross motions refer to stars situated in a definite small portion of the sky, and, owing to the effects of star-streaming, these will be different from the mean values for the whole sky. If we suppose that the Lick results are proportional to the mean radius, with respect to solid angle, of Schwarzschild's ellipsoid, and that the present results are proportional to the radius to the pole, a rough comparison between them can be made. Taking the position of Kapteyn's preferential vertex as R.A. 90°, Dec. + 13°, and using the dimensions of the ellipsoid as calculated above, I find that, with these assumptions, for the groups 2, 3, 4, 5 respectively in Table VI., the Lick results should be the larger in the ratios 1·08, 1·16, 1·12, and 1·07. The mean τ for the group B0–B7, resting as it does upon only eleven stars, cannot be

* For Groups G, K, M respectively.

compared with the Lick values: but for the other groups the values found in the present paper, when corrected so as to give the mean for the whole sky, will give results which are larger than Campbell's except for the group G6–M, where the two results become equal. The differences are not great in view of the facts that different stars have been used in the two determinations, and also that if a value smaller than 19·5 were used for the solar velocity, the velocities as here determined would be proportionately reduced. Since the differences are systematic, an explanation could also be given by supposing that the fainter stars are travelling quicker than the brighter ones, but not much stress should be laid upon this explanation. Campbell, indeed, thought at first that he had obtained indications that the fainter stars are travelling the quicker, but later results have not tended to confirm this statement.

The present investigation gives additional confirmation to the now well-established fact that the average stellar velocity increases progressively with advancing spectral type. Combining this result with the above determinations of the mean parallaxes, it appears that the small proper-motions of the stars in Group 1 result from a combination of great distance with small velocities. The stars in Group 2 are on the average nearer and move faster. Group 3 stars are still nearer and have still larger velocities, which combine to give their larger proper motions. In Groups 4 and 5 the effect of greatly increased distance more than counterbalances the effect of their greater intrinsic velocities, and reduces their apparent velocities.

In figs. 1–4 (Plates 6 and 7), the centennial proper motions of the stars of Groups 2–5 respectively have been graphically depicted, the abscissæ and ordinates representing the apical and cross-motion components. The apex of the solar motion is in the direction marked 0°, the antapex being towards the direction of 180°. If all the stars in this area of the sky were conceived as placed together at the pole, then at the end of a century they would form a more or less scattered group, each having travelled to a point whose coordinates are given by the components of its centennial proper motion (gravitational forces, of course, being supposed not to come into play). The diagrams therefore represent approximately what would be the distribution of the stars of the various groups at the end of a century. The eleven stars of Group 1 have been included in fig. 1, and denoted by a +. The differences in the nature of the proper motions of the several groups are strikingly shown. The stars of Group 2 form a very compact group; as in all the figures, owing to the solar motion, many more appear to have moved towards the antapex than in the opposite direction. There is a distinct tendency to concentrate in this direction about the line joining the apex and antapex, indicating that in a large number of cases the proper motion is mainly parallactic in origin. The Group 3 stars are much more scattered than those of any of the other groups, a large majority of the big proper motions being

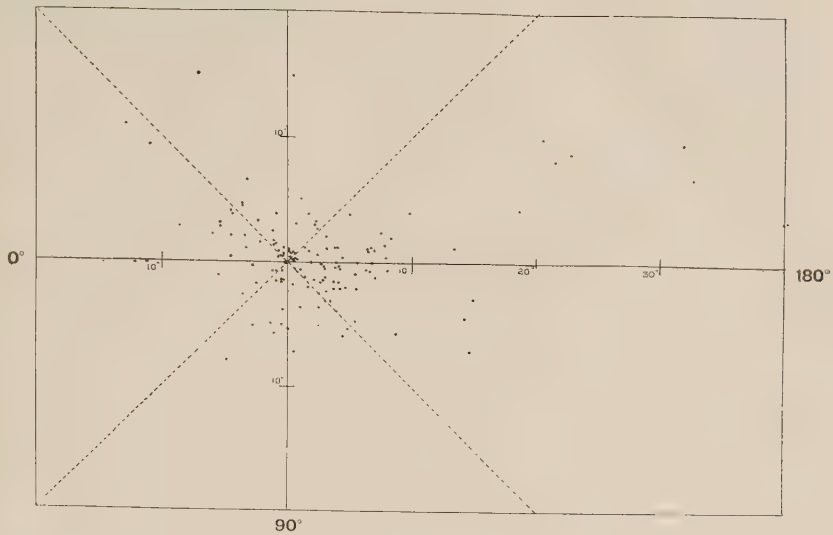


Fig. 3.—Proper Motions of Group G0–G5.

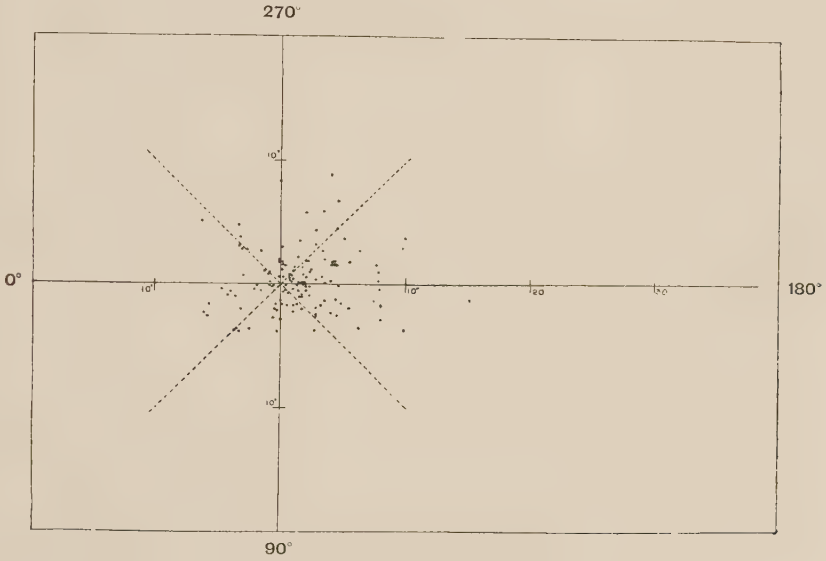


Fig. 4.—Proper Motions of Group G6–M.

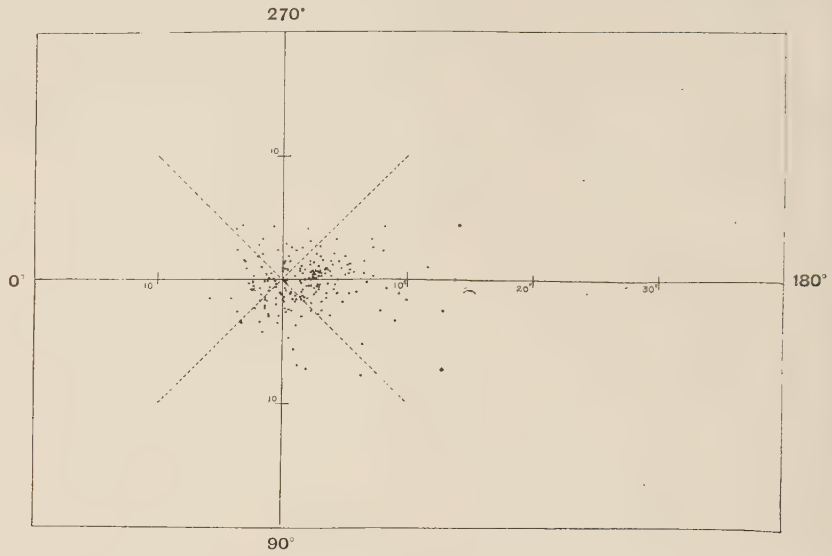


Fig. 1.—Proper Motions of Group $\frac{\pi}{\mu}$ B0–A4.

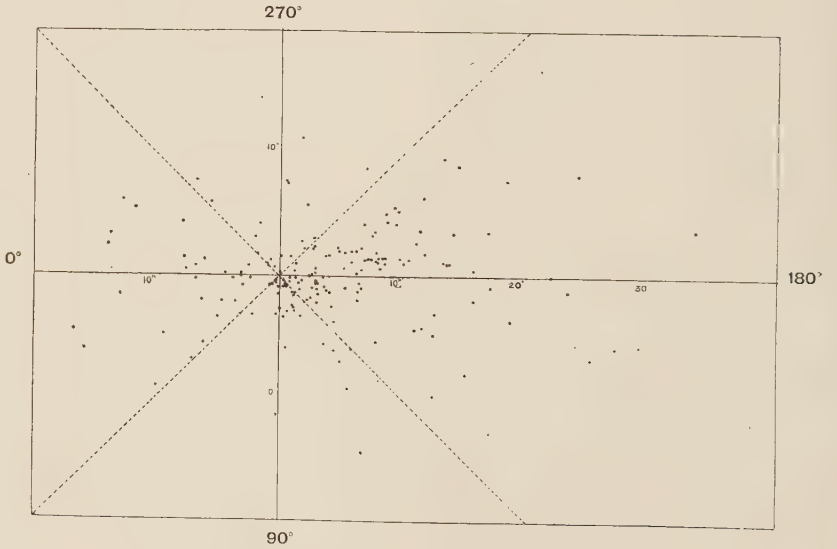


Fig. 2.—Proper Motions of Group A5–F9.

included in it. In Group 4 most of the stars are clustered fairly uniformly about the origin, but there are a few which have moved a large distance: this group seems to consist of two classes, the proper motions of one class being small and of the other large. In Group 5, which closely resembles Group 2, except that the distribution in the four quadrants is more uniform, there are no very large motions. The numbers of stars of the various groups in the four quadrants formed by the lines inclined at $\pm 45°$ to the line from the apex to the antapex are given in Table VII., the quadrants 1, 2, 3, 4 being respectively those whose bisectors are in the directions 0°, 90°, 180°, and 270°.

Table VII.

Group Limits.	1.	2.	3.	4.	Total.
B0–A4	29	43	107	20	199
A5–F9	33	35	102	17	187
G0–G5	25	22	67	27	141
G6–M	22	22	71	25	140

There is a great similarity in the quadrantal distribution of the first two of the above groups, and also in the second two. In each case a large majority of the stars are in the quadrant containing the antapex. In the first two groups there is a slight secondary maximum in the quadrant about the 90° direction, which is possibly due to star-streaming: but this is uncertain, as the maximum is not very clearly defined. In the second two groups the diagrams show practically no evidence of streaming. To make this apparent many more stars would need to be used.

It has been mentioned above that Dr. Dyson and Professor Eddington have shown that a law of distribution of the stars in space such that $2k^2re^{-k^2r^2}dr$ is the proportion between distances r and $r+dr$ is in good agreement with observation. This law requires that the proportion of cross proper motions between the limits τ and $\tau+d\tau$ be

$$\frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-\frac{3}{2}},$$

where $a=k/h$, h and k being constants occurring in the exponential terms $e^{-h^2v^2}$, $e^{-k^2r^2}$ in the laws of velocity and spatial distribution respectively.

It seemed of interest to ascertain how closely this law was in agreement with the results obtained above for the different types. The total number of stars in each group is somewhat small for this purpose, but yet quite sufficient to form an opinion as to the qualitative agreement of the formula with the proper-motion distribution, and to obtain an approximate value for the constant

a, which may easily be shown to equal the mean value of the cross proper motions. In fact this mean value is

$$\int_{-\infty}^{+\infty}\int_{0}^{\infty} \frac{h}{\sqrt{\pi}} \cdot e^{-h^2v^2} dv \,.\, 2k^2 r e^{-k^2r^2} dr \cdot \frac{v}{r} = \frac{k}{h} = a.$$

If the number of proper motions between τ and $\tau + d\tau$ be

$$N \frac{d\tau}{a}\left(1 + \frac{\tau^2}{a^2}\right)^{-\frac{3}{2}},$$

where N is the total number of stars, then the number between the limits 0 and τ is

$$\frac{N\tau}{\sqrt{a^2 + \tau^2}}.$$

If this law of distribution be the true one, it should be possible, by suitable choice of the constant a, to make the numbers calculated from this formula agree with the numbers read off from the smoothed curve drawn to fit the observations as well as possible, after the latter have been corrected for the effects of a probable error of 0″·75 per century.

It was found that for the groups A5–F9, and G0–G5 a very good agreement was obtained. The comparison between the numbers as read off from the smoothed curves and the numbers calculated from the above formula are given in Table VIII. It will be seen that the formula represents well the distribution of the large proper motions, but that it gives rather fewer small ones than are actually found, but in this portion of the curve small discrepancies do not possess a very great significance.

TABLE VIII.

Value of τ.	Group A5–F9. Value of a=3″·46. No. of Stars.			Group G0–G5. Value of a=3″·01. No. of Stars.		
	Obs.	Calc.	Diff.	Obs.	Calc.	Diff.
1	54	52	+ 2	50	44	+6
2	104	93½	+10½	77½	77½	0
3	125	123	+ 2	96½	99	−2½
4	139	141	− 2	110	113	−3
5	150	153	− 3	119	121	−2
6	159½	162	− 2½	125½	126	− ½
7	166½	166½	0	129	129½	− ½
8	171½	171½	0	132	132	0
9	175	174½	+ ½	133½	133½	0
∞	187	187	0	141	141	0

These values of a were chosen so as to give the best agreement for the large proper motions: the formula then gives fewer small values than are observed and so these values of a are larger than the values of the mean cross motion. By taking rather smaller values of a, the general agreement would be improved, but for the large values of τ would not be quite so good. These figures, however, are sufficient to show that the assumed law of distribution is closely approximated to in these two groups.

Assuming, therefore, this law of distribution, the coefficient k can be determined, for the mean parallax is given by

$$\pi = \int_0^\infty 2k^2 e^{-k^2r^2} dr$$
$$= k\sqrt{\pi}.$$

Whence for the group A5–F9 $k = \cdot00588$, and for the group Go–G5 $k = \cdot00406$.

The percentages of these stars at different distances can accordingly be deduced, and are given in Table IX., the unit of distance corresponding to a parallax of 1″.

TABLE IX.

Distance.	Parallax.	Percentage.	
		A5–F9.	Go–G5.
0–20	>″·050	1·3	0·8
20–40	″ 050–″·025	3·9	2·0
40–100	·025–·010	24·1	12·0
100–200	·010–·005	45·6	33·4
200–400	·005–·0025	24·7	42·7
400–600	.0025–·0017	...	6·8
>400	<″·0025	0·4	...
>600	<·0017	...	0·3

Reckoning to the nearest integer, these results indicate that 2 stars of the A5–F9 group, and 1 of the Go–G5 group have a parallax >″·050, and that 10 of the former and 4 of the latter have parallaxes >″·025, whilst practically all of the former have parallaxes >″·0025, and all of the latter >″·0017. Of the A5–F9 stars here considered about 94 per cent. and of the Go–G5 stars about 88 per cent. have parallaxes between ″·025 and ″·0025.

For the other two groups, B8–A4 and G6–M, the agreement between the formula and the observations was not so good: the discrepancies were greatest for the former group, and it is significant that Professor Eddington * found discrepancies between the calculated distribution of A type stars based upon the law $Are^{-k^2r^2}dr$ and the observed distribution. An attempt was made to see whether

* *M.N., R.A.S.*, vol. lxxiii., 1913, p. 358.

the more general law $Ar^{\lambda}e^{-k^2r^2}dr$ could, by suitable choice of λ, be brought into agreement with the observations. According to this law, the number of cross proper motions between τ and $\tau + d\tau$ is proportional to

$$\frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-\frac{\lambda}{2}-1}.$$

Little improvement, however, was thus obtained.

As an example of the discordance, the observed and calculated values for the group G6–M are given in Table X., the value of a having been chosen so as to make what, on the whole, appeared the best fit. Had the agreement been improved for the large values of τ, the discrepancy would have been largely increased for the small values.

TABLE X.

Value of $a = 1{\cdot}80$.

τ.	No. of Stars.		Difference.
	Observed.	Calculated.	
1	53	67½	− 14½
2	96½	104	− 7½
3	118	120	− 2
4	128	128	0
5	133	132	+ 1
6	135	134	+ 1
7	136½	135½	+ 1

*Galactic Co-ordinates.** By O. R. Walkey.

In view of the growing recognition of the galactic plane as the true reference for star positions in stellar problems, the accompanying table (computed some years ago) is given of the galactic co-ordinates for the intersection of every hour-meridian of Right Ascension with each 10-degree parallel of Declination. The co-ordinates are referred to the great circle having its north pole in R.A. $12^h\ 47^m$; decl. $+27°$ (or south pole in R.A. $0^h\ 47^m$; decl. $-27°$). A similar table has appeared in *Harvard Annals*, vol. lvi., part i., for each 10-degree interval with the north pole in R.A. $12^h\ 40^m$; decl. $+28°$, or nearly 2 degrees north-west of the one here adopted. The Harvard pole, however, was an approximate one, taken, apparently for simplicity's sake, exactly on the $12^h\ 40^m$ (190°) R.A. meridian, as near enough for the purpose in question.

Dr. A. Marth published in 1872–3 (*Monthly Notices*, vols. xxxiii., xxxiv.), the galactic co-ordinates of all individual stars down to a certain magnitude, using a different pole. Professor J. C. Kapteyn and others have adopted still different poles. Considering the rapidly advancing studies in matters stellar, it has become highly desirable that an agreement be now arrived at by astronomers upon some great circle which is to serve henceforth as a common galactic plane of reference for all star positions.

The pole here given lies in the mean of the two positions deduced from careful computation by the late Professor Newcomb (assisted by Mr. W. T. Carrigan) for the main galactic stream, including and excluding respectively the "arm" in Cygnus and Aquila. The adopted mean therefore includes this arm, with half "weight," and, curiously enough, coincides exactly with the position assigned by Sir J. Herschel (as quoted by Professor Young) and given generally in text-books.

Until a better be found, this pole, together with the table based thereupon, is here given as representing the great circle suggested for general adoption in order to avoid certain confusion or needless complication in the near future. The positions are tabulated to the nearest tenth of a degree, as being sufficient for plotting on all ordinary charts. The nearest approaches (or summits of the curves) of the four R.A. hour-meridians passing near to each galactic pole are given as a probable aid to plotting—especially on such a projection as the Mercator, when so used.

In the event of the general adoption of this (or another) great circle as the common reference, the corresponding galactic co-ordinates can be computed to the nearest minute of arc and tabulated for (say) every 20 minutes of R.A. and 5 degrees of declination when such general adoption makes it worth while.

* As the hitherto published tables of Galactic Co-ordinates are not very widely accessible, it is thought that the publication here of Mr. Walkey's table will meet a general need.—Secs.

FOR EACH HOUR OF RIGHT ASCENSION AND TEN DEGREES OF DECLINATION. REFERRED

R.A.	I.		II.		III.		IV.		V.		VI.	
Decl.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.
+90°	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°
+80	90·6	+17·0	93·3	+17·5	95·8	+18·5	97·9	+20·1	99·6	+22·2	100·8	+24·
+70	91·1	+ 7·0	96·1	+ 7·9	101·0	+ 9·9	105·2	+12·9	108·6	+16·8	111·1	+21·
+60	91·6	- 3·0	99·0	1·7	105·9	+ 1·2	112·0	+ 5·5	117·1	+11·1	120·9	+17·
+50	92·1	-13·0	101·8	-11·3	110·8	- 7·5	118·7	- 1·9	125·2	+ 5·2	130·3	+13·
+40	92·7	-22·9	104·9	-20·9	115·9	-16·2	125·4	- 9·4	133·2	- 0·9	139·4	+ 8·
+30	93·4	-32·9	108·3	-30·4	121·5	-24·7	132·4	-16·7	141·2	- 6·9	148·2	+ 4·
+20	94·2	-42·9	112·5	-39·8	127·9	-33·0	140·0	-23·7	149·4	-12·8	156·9	- 0·
+10	95·3	-52·9	118·0	-49·0	135·6	-40·9	148·4	-30·4	158·0	-18·4	165·7	- 5·
0	97·1	-62·8	126·0	-57·8	145·3	-48·2	157·9	-36·4	167·1	-23·6	174·6	-10·
-10	100·8	-72·7	138·9	-65·8	157·9	-54·4	169·0	-41·5	177·0	-28·3	183·6	-14·
-20	113·7	-82·4	161·0	-71·9	174·1	-58·8	181·6	-45·5	187·6	-32·1	193·3	-19·
-30	227·2	-85·9	194·9	-73·7	193·8	-60·7	195·9	-47·8	199·1	-35·0	203·3	-22·
-40	259·1	-76·7	225·3	-70·1	213·8	-59·6	210·8	-48·3	211·3	-36·8	213·8	-25·
-50	264·7	-66·9	243·6	-63·1	231·2	-55·8	225·5	-46·8	223·8	-37·3	224·7	-27·
-60	267·0	-56·9	254·3	-54·7	244·8	-50·0	238·5	-44·4	236·3	-36·4	236·0	-28·
-70	268·4	-47·0	261·2	-45·7	255·1	-43·0	250·8	-39·0	248·3	-34·3	247·4	-29·
-80	269·3	-37·0	266·1	-36·4	263·3	-35·2	261·1	-33·4	259·6	-31·1	258·8	-28·
-90	270·0	-27·0	270·0	-27·0	270·0	-27·0	270·0	-27·0	270·0	-27·0	270·0	-27·

R.A.	XIII.		XIV.		XV.		XVI.		XVII.		XVIII.	
Decl.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.
+90°	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°
+80	89·3	+37·0	86·1	+36·4	83·3	+35·2	81·1	+33·4	79·6	+31·1	78·8	+28·
+70	88·4	+47·0	81·2	+45·7	75·1	+43·0	70·8	+39·0	68·3	+34·3	67·4	+29·
+60	87·0	+56·9	74·3	+54·7	64·8	+50·0	58·5	+44·4	56·3	+36·4	56·0	+28·
+50	84·7	+66·9	63·6	+63·1	51·2	+55·8	45·5	+46·8	43·8	+37·3	44·7	+27·
+40	79·1	+76·7	45·3	+70·1	33·8	+59·6	30·8	+48·3	31·3	+36·8	33·8	+25·
+30	47·2	+85·9	14·9	+73·7	13·8	+60·7	15·9	+47·8	19·1	+35·0	23·3	+22·
+20	293·7	+82·4	341·0	+71·9	354·1	+58·8	1·6	+45·5	7·6	+32·1	13·3	+19·
+10	280·8	+72·7	318·9	+65·8	337·9	+54·4	349·0	+41·5	357·0	+28·3	3·6	+14·
0	277·1	+62·8	306·0	+57·8	325·3	+48·2	337·9	+36·4	347·1	+23·6	354·6	+10·
-10	275·3	+52·9	298·0	+49·0	315·6	+40·9	328·4	+30·4	338·0	+18·4	345·7	+ 5·
-20	274·2	+42·9	292·5	+39·8	307·9	+33·0	320·0	+23·7	329·4	+12·8	336·9	+ 0·
-30	273·4	+32·9	288·3	+30·4	301·5	+24·7	312·4	+16·7	321·2	+ 6·9	328·2	- 4·
-40	272·7	+22·9	284·9	+20·9	295·9	+16·2	305·4	+ 9·4	313·2	+ 0·9	319·4	- 8·
-50	272·1	+13·0	281·8	+11·3	290·8	+ 7·5	298·7	+ 1·9	305·2	- 5·2	310·3	-13·
-60	271·6	+ 3·0	279·0	+ 1·7	285·9	- 1·2	292·0	- 5·5	297·1	-11·1	300·9	-17·
-70	271·1	- 7·0	276·1	- 7·9	281·0	- 9·9	285·2	-12·9	288·6	-16·8	291·1	-21·
-80	270·6	-17·0	273·3	-17·5	275·8	-18·5	277·9	-20·1	279·6	-22·2	280·8	-24·
-90	270·0	-27·0	270·0	-27·0	270·0	-27·0	270·0	-27·0	270·0	-27·0	270·0	-27·

NEAREST APPROACHES TO GALACTIC NORTH POLE.

XI.		XII.		XIII.		XIV.	
Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat
167°1	+66°4	174°6	+79°5	1°5	+87°1	8°5	+73

HE G.N.P. IN R.A. $12^{H}\ 47^{M}$; DECL. $+27°$ AND G.S.P. IN R.A. $0^{H}\ 47^{M}$ DECL. $-27°$.

VII.		VIII.		IX.		X.		XI.		XII.		R.A.
Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Decl.
90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	+90°
101·2	+27·1	100·9	+29·7	99·9	+32·1	98·0	+34·2	95·5	+35·8	92·5	+36·8	+80
112·4	+26·4	112·4	+31·5	110·8	+36·4	107·5	+40·8	102·4	+44·3	95·8	+46·5	+70
123·3	+24·7	124·1	+32·2	122·9	+39·6	118·9	+46·5	111·6	+52·3	100·5	+56·0	+60
133·9	+22·3	135·9	+31·8	135·8	+41·4	132·7	+50·8	124·4	+59·2	108·3	+65·3	+50
144·1	+19·3	147·5	+30·4	149·2	+41·8	148·5	+53·2	142·8	+64·3	123·9	+73·8	+40
153·9	+15·7	158·6	+27·9	162·4	+40·6	165·2	+53·4	166·3	+66·3	160·7	+79·2	+30
163·4	+11·7	169·1	+24·7	174·8	+37·9	181·1	+51·3	190·2	+64·6	210·5	+77·2	+20
172·5	+ 7·4	179·1	+20·7	186·3	+34·0	195·2	+47·2	208·8	+59·6	234·6	+69·7	+10
181·5	+ 2·9	188·5	+16·2	196·6	+29·2	206·9	+41·7	222·0	+52·7	245·4	+60·7	0
190·4	− 1·7	197·5	+11·3	205·9	+23·7	216·7	+35·2	231·3	+44·8	251·3	+51·3	−10
199·3	− 6·2	206·2	+ 6·1	214·4	+17·7	224·9	+28·0	238·3	+36·3	255·2	+41·7	−20
208·4	−10·6	214·7	+ 0·8	222·4	+11·3	232·0	+20·4	243·9	+27·5	258·0	+31·9	−30
217·8	−14·7	223·1	− 4·5	230·0	+ 4·7	238·5	+12·6	248·7	+18·5	260·3	+22·1	−40
227·5	−18·4	231·7	− 9·7	237·5	− 1·9	244·6	+ 4·6	252·9	+ 9·4	262·3	+12·3	−50
237·5	−21·6	240·6	−14·7	245·0	− 8·6	250·5	− 3·5	257·0	+ 0·3	264·2	+ 2·5	−60
248·0	−24·2	249·9	−19·3	252·8	−15·0	256·6	−11·5	261·0	− 8·9	266·0	− 7·4	−70
258·9	−26·0	259·6	−23·5	261·0	−21·2	263·0	−19·4	265·3	−18·0	267·9	−17·2	−80
270·0	−27·0	270·0	−27·0	270·0	−27·0	270·0	−27·0	270·0	−27·0	270·0	−27·0	−90

XIX.		XX.		XXI.		XXII.		XXIII.		XXIV.		R.A.
Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Decl.
90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	90°0	+27°0	+90°
78·9	+26·0	79·6	+23·5	81·0	+21·2	83·0	+19·4	85·3	+18·0	87·9	+17·2	+80
68·0	+24·2	69·9	+19·3	72·8	+15·0	76·6	+11·5	81·0	+ 8·9	86·0	+ 7·4	+70
57·5	+21·6	60·6	+14·7	65·0	+ 8·6	70·5	+ 3·5	77·0	− 0·3	84·2	− 2·5	+60
47·5	+18·4	51·7	+ 9·7	57·5	+ 1·9	64·6	− 4·6	72·9	− 9·4	82·3	−12·3	+50
37·8	+14·7	43·1	+ 4·5	50·0	− 4·7	58·5	−12·6	68·7	−18·5	80·3	−22·1	+40
28·4	+10·6	34·7	− 0·8	42·4	−11·3	52·0	−20·4	63·9	−27·5	78·0	−31·9	+30
19·3	+ 6·2	26·2	− 6·1	34·4	−17·7	44·9	−28·0	58·3	−36·3	75·2	−41·7	+20
10·4	+ 1·7	17·5	−11·3	25·9	−23·7	36·7	−35·2	51·3	−44·8	71·3	−51·3	+10
1·5	− 2·9	8·5	−16·2	16·6	−29·2	26·9	−41·7	42·0	−52·7	65·4	−60·7	0
352·5	− 7·4	359·1	−20·7	6·3	−34·0	15·2	−47·2	28·8	−59·6	54·6	−69·7	−10
343·4	−11·7	349·1	−24·7	354·8	−37·9	1·1	−51·3	10·2	−64·6	30·5	−77·2	−20
333·9	−15·7	338·6	−27·9	342·4	−40·6	345·2	−53·4	346·3	−66·3	340·7	−79·2	−30
324·1	−19·3	327·5	−30·4	329·2	−41·8	328·5	−53·2	322·8	−64·3	303·9	−73·8	−40
313·9	−22·3	315·9	−31·8	315·8	−41·4	312·7	−50·8	304·4	−59·2	288·3	−65·3	−50
303·3	−24·7	304·1	−32·2	302·9	−39·6	298·9	−46·5	291·6	−52·3	280·5	−56·0	−60
292·4	−26·4	292·4	−31·5	290·8	−36·4	287·5	−40·8	282·4	−44·3	275·8	−46·5	−70
281·2	−27·1	280·9	−29·7	279·9	−32·1	278·0	−34·2	275·5	−35·8	272·5	−36·8	−80
270·0	−27·0	270·0	−27·0	270·0	−27·0	270·0	−27·0	270·0	−27·0	270·0	−27·0	−90

NEAREST APPROACHES TO GALACTIC SOUTH POLE.

XXIII.		XXIV.		I.		II.	
Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.
347°1	−66°4	354°6	−79°5	181°5	−87°1	188°5	−73°8

Hydrogen and the Primary Constituents of Nebulæ. By J. W. Nicholson, M.A., D.Sc., Professor of Mathematics in the University of London.

The series relations which connect the lines in the spectra of Wolf-Rayet stars, communicated recently to the Society,* appear to throw some light on the nature of series spectra in general. Apparently the greater part of the Wolf-Rayet spectrum can be grouped into the single formula

$$\lambda = \frac{2\lambda_0}{1+nd}\left(\frac{m^2}{m^2 - q^2/4\omega^2}, \frac{m^2}{m^2-1}\right),$$

where λ_0 is the wave-length of a line given by a simple-ring atom in the nebulæ—the atom of neutral nebulium,—and q/ω is any one of the period ratios appropriate to that atom. The integers m and n can have any positive values, and it is supposed that a change of n denotes a transition to a substance of different internal constitution, whereas, for a given value of n, all the lines corresponding to a variable m arise from the same atom. The second part of the formula, with the factor $m^2/(m^2-1)$, is not essentially different from the first, for one of the values of q/ω for any simple-ring system is always unity, and the factor 4 signifies that for this particular case half-integer values of m are admissible.

Balmer's formula for hydrogen is of the same type, for it may be written

$$\lambda = 3647\frac{m^2}{m^2-1},$$

where integers and half-integers give admissible values of m. This similarity appears to imply, of necessity, a similarity in mode of production of the series, especially since hydrogen invariably accompanies the substances which give rise to the preceding formulæ. In other words, hydrogen must mark a stage in the evolution of matter from a certain simple-ring system. This must, in any case, be a very early stage, for hydrogen is an important constituent of all nebulæ showing the line $\lambda 5007$, and therefore still preserving the simple-ring system of nebulium. If this conclusion be valid, it is possible to obtain information as to the simple-ring system from which hydrogen is evolved. If we find that it is a system consisting of a nucleus ne of positive electricity, surrounded by n electrons in steady rotation, then it is possible to assert that the atom of hydrogen consists of this system modified in such a way as to preserve all the values of q/ω—or, in other words, that this primary system can only have been modified as regards the structure of its nucleus, which must still attract all the electrons according to the law of inverse square and preserve

* *Monthly Notices*, December 1913.

its charge unaltered. A change in the nature of the bond which links the nucleus to each of the electrons may also occur, but not in such a way as to disturb the force acting on one of the electrons.

Whatever the simple system from which it is derived, hydrogen should also give series

$$\lambda = 3647 \frac{m^2}{m^2 - q^2/4\omega^2},$$

where q/ω can have any of the values appropriate to that system. We proceed to search for such series. The most important vibration of a simple atom, except $q = \omega$, is given by

$$q/\omega = \left(\frac{\nu}{\nu - \frac{1}{4}S_n}\right)^{\frac{1}{2}} \left(1 + \frac{n}{2}\frac{m}{M}\right) \quad (n \neq 1).$$

This is the principal vibration, of class zero, transverse to its plane. The strength of the nucleus is νe, and m and M are the masses of electron and nucleus. There are n electrons, and

$$S_n = \sum_{r=1}^{n-1} \operatorname{cosec} \frac{r\pi}{n}.$$

It is known, moreover, that the retention of the term in m/M only introduces a difference in q/ω of one or two parts in 1000, so that in a preliminary survey the term may be ignored. In formulæ of the present type the difference introduced in λ is even less.

We shall suppose, in the first place, that the hydrogen spectrum is derived from a neutral atom. The following is a summary of the results of calculation :—

(1)
$$\nu = n = 2, \quad S_n = 1$$
$$q^2/\omega^2 = 8/7$$
$$\lambda = 3647 \frac{m^2}{m^2 - \frac{2}{7}}.$$

The members of this series are :—

$m = 1$,	$m = 1\frac{1}{2}$,	$m = 2$,	$m = 2\frac{1}{2}$,	$m = 3$.
5105·8,	4177·5,	3927·5,	3822,	3767.

The possible case of half-integers is included for completeness, but none of these lines is close to any important line of unknown origin in any astrophysical spectrum. They also do not correspond to lines in the terrestrial secondary spectrum of hydrogen.

(2)
$$\nu = n = 3, \quad S_n = 4/\sqrt{3}$$
$$q^2/\omega^2 = 1{\cdot}2383$$
$$\lambda = 3647 \frac{m^2}{m^2 - 1{\cdot}2383}$$

(including half-integers) with members

$m = 2$,	$m = 3$,	$m = 4$,	$m = 5$,	$m = 6$,	$m = 7$.
5282,	4228·7,	3953,	3837·3,	3777,	3742.

The first two are near Wolf-Rayet lines already explained in a different way, essential to the whole scheme. But otherwise these lines are unknown. They should be strong if due to hydrogen.

(3) $$\nu = n = 4, \quad S_n = 1 + 2\sqrt{2}$$
$$q^2/\omega^2 = 1{\cdot}3145$$

(the nebulium atom, which is *prima facie* unlikely),

$$\lambda = 3647\frac{m^2}{m^2 - 1{\cdot}3145},$$

with members

$m=2$,	$m=3$,	$m=4$,	$m=5$,	$m=6$.
5433,	4271,	3975,	3850,	3786.

Except that the third and fourth are weak Orion lines, this spectrum does not seem to exist.

Further examination leads to the same conclusion regarding other systems; but it is not actually necessary to proceed further, for the mass of the nucleus in nebulium, deduced from its spectrum, is greater than the atomic weight of hydrogen. Whatever form the evolution in the stars may take, it can hardly in the normal case be in the direction of decreasing atomic weight.

Thus the simple-ring system which can evolve into hydrogen can only have *one* outer electron, and it is necessary to show that the formula $q/\omega = 1$ holds rigorously for such a system, in all its vibrations. For the Balmer formula is extremely accurate, and if it is derived after the manner of the Wolf-Rayet formula, the relation $q/\omega = 1$ must be exact, and the ordinary formula for transverse vibrations must fail when $n = 1$.

Recent atomic theory denies the complete applicability of Newtonian dynamics to the vibrations of an atom in its plane, while perhaps retaining the ordinary frequency $q = \omega$. But it is agreed that for vibrations perpendicular to the plane, Newtonian dynamics can be employed.*

When there is more than one electron in the atom, the nucleus is situated at the common centre of gravity of the system during steady motion, and the ordinary formula

$$q^2 = \omega^2\left(\frac{\nu}{\nu - \frac{1}{4}S_n}\right)^{\frac{1}{2}}\left(1 + \frac{n}{2}\frac{m}{M}\right)$$

applies to the principal transverse vibration of class zero. When there is only one electron, the nucleus also rotates in steady motion, and the period equation changes. If the nucleus did not move, any transverse disturbance of the electron would not strictly be an oscillation, but only a shift of the plane of the orbit. But with a free nucleus, a transverse oscillation of the system is possible, in which the common centre of gravity remains at rest.

* Bohr, *Phil. Mag.*, July 1913.

In the figure let O be the fixed centre, and a, b the distances, in steady motion with angular velocity ω, of the positive and negative charges from it. For generality the charges may be taken unequal, the nucleus having strength ve, and mass M, as against e and m for the electron. Then in steady motion,

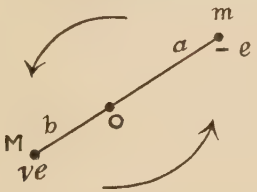
$$ma\omega^2 = \mathrm{M}b\omega^2 = \frac{ve^2}{(a+b)^2},$$

or, eliminating b,

$$\omega^2 = \frac{ve^2}{ma^3}\left(1 + \frac{m}{\mathrm{M}}\right)^{-2}.$$

Let (z, ζ) be transverse displacements of electron and nucleus in an oscillation. Then

$$mz + \mathrm{M}\zeta = 0.$$



The distance apart is $[(a+b)^2 + (z-\zeta)^2]^{\frac{1}{2}}$, and the equations of motion become

$$m\ddot{z} = -\frac{(z-\zeta)ve^2}{(a+b)^3} = -\mathrm{M}\ddot{\zeta},$$

or if z is proportional to $\cos qt$,

$$mq^2 = \left(1 + \frac{m}{\mathrm{M}}\right)(a+b)^{-3}ve^2 = \frac{ve^2}{a^3}\left(1 + \frac{m}{\mathrm{M}}\right)^{-2} = m\omega^2,$$

or $q = \omega$. In the case of one electron, therefore, the mass of the nucleus disappears from the period equation, and the *only* frequency of the atom is $q/2\pi$, whatever the strength of the nucleus. Thus there should be *no other series of limit* 3647 for the hydrogen-atom if it contains only one outer electron, and the fact that no other such series exists for hydrogen, though it does for the Wolf-Rayet elements, leads to the conclusion that hydrogen does possess only one outer electron. This conclusion has been obtained by other investigators from quite different lines of reasoning. Rutherford has obtained it as the only explanation of the scattering effect of hydrogen on radioactive particles, and Bohr has founded a theory of the production of Balmer's formula on the same hypothesis. Arrived at in such different ways, the conclusion now seems certain. But it does not imply that the hydrogen atom is a simple-ring system in the ordinary sense, analogous to nebulium. We maintain that nebulium itself, for example, cannot show series, but that the interior of the atom is

modified in some way before series appear. Hydrogen is only an evolution product from the simple system with a unit nucleus, and probably an early, in fact the first product, from its early appearance in nebulæ.

Confining our attention now to the simple-ring system from which the hydrogen atom is apparently derived, the first necessity which arises is the selection of a suitable name. We shall call it *protohydrogen*, for although this nomenclature, when used earlier by Lockyer, has a different significance, its use is appropriate here, since no simpler form of hydrogen can be imagined except this simple-ring system from which it is immediately derived by some internal change. Protohydrogen can only give one spectrum-line whose wave-length will be calculated later.

Let us consider the angular momentum of such an atom in steady motion, λ being the wave-length emitted during an oscillation, so that if C is the velocity of light,

$$\lambda = 2\pi C/\omega.$$

The total angular momentum is

$$ma^2\omega + Mb^2\omega = m\left(1 + \frac{m}{M}\right)a^2\omega,$$

where

$$ma^2\omega = m\omega\left(\frac{e^2}{m\omega^2}\right)^{\frac{2}{3}}\left(1 + \frac{m}{M}\right)^{-}$$

$$= m^{\frac{1}{3}}e^{\frac{4}{3}}\left(\frac{\lambda}{2\pi C}\right)^{\frac{1}{3}}\left(1 + \frac{m}{M}\right)^{-\frac{1}{3}}.$$

If A is the *total* angular momentum,

$$A^3 = \frac{me^4}{2\pi C}\lambda\left(1 + \frac{m}{M}\right)^{-1};$$

and if B is the angular momentum of electron,

$$B^3 = \frac{me^4}{2\pi C}\lambda\left(1 + \frac{m}{M}\right)^{-4},$$

whereas for an atom with more than one electron,

$$A^3 = B^3 = \frac{me^4}{2\pi C}\lambda(\nu - \tfrac{1}{4}S_n)^2$$

independent of the mass ratio.

We have seen in the case of protofluorine that there is a simple relation between the angular momenta of simple-ring systems and Planck's constant, which determines the absolute dimensions of the atoms, and which may be employed in predicting wave-lengths for some systems, given those of others. It cannot be applied immediately to systems with only one electron, on account of the difference in the law connecting angular momentum and principal wave-length in this case. But since m/M can only make a difference of a few tenth metres in a wave-length, it can be applied approximately. It is not, of course, possible to say *which*

angular momentum is concerned here, for when the positive electricity is not at rest, it may be the angular momentum of the whole atom, of the electron only, or even other alternatives. But it happens that the whole difference between the alternatives only leads to a change of a few tenth metres in A.

Possible Extension of the Hydrogen Spectrum.

It was suggested in an earlier paper,* that certain lines in the Wolf-Rayet stars and elsewhere, satisfying the general formula

$$\lambda = 3647 \frac{m^2}{m^2 - n^2}$$

where m and n are integers, indicated the fact that this was a more complete law than Balmer's for the hydrogen spectrum. For example, it includes the Pickering lines also. Some further considerations may now be given in support of this suggestion. If it is possible that a single electron bound in an atom can describe uniformly a circle about the nucleus without loss of energy by radiation—a possibility which, in the light of recent theory of atomic energy, may now be regarded almost as a certainty,—then it is equally possible for the electron to describe an ellipse. This elliptic path, being described about the focus under the action of a central force, would preserve the necessary constancy of angular momentum during the undisturbed motion. Any small disturbance can be analysed into a set of harmonic disturbances of frequencies $n\omega/2\pi$, where ω is the mean angular velocity in the undisturbed orbit, and n is any integer. If the atom can itself accomplish this resolution, it gives a set of frequencies according to the equation $q = n\omega$, and the formula

$$\lambda = \lambda_0 \frac{m^2}{m^2 - q^2/4\omega^2}$$

leads to the general result

$$\lambda = \lambda_0 \frac{m^2}{m^2 - n^2},$$

where m and n are any integers. These vibrations should become less important as n increases, and comparatively insignificant when the orbit is nearly circular. It is at least possible that this is the actual source of the Pickering series, quite apart from its possible origin from helium in Bohr's theory, the two origins not being mutually exclusive. If such an origin is proposed, it is necessary only to assume that the normal orbit of the electron is circular, but that violent agencies are at work in celestial spectra which can render it elliptical. Bohr's theory goes further in giving also Fowler's series and the Ritz series in the infra-red, but it does not give the extra lines for $n = 3$ and $n = 4$ without the supposition that they are due to lithium and beryllium with a very large positive charge, and there is no other evidence for the presence of these

* *Monthly Notices*, February 1913.

elements in the stars in question. Without a theory of the origin of the Wolf-Rayet formula, the question must be left open, but it is necessary to give this indication that Pickering's series may still be a hydrogen series, even if it is a helium series also. The Wolf-Rayet formula alone does not, of course, take *Principal* series into its scope, and therefore gives no evidence as to whether Fowler's series can also belong to hydrogen or not.

The Angular Momenta of Simple-ring Systems.

We proceed now to a brief account of the law which regulates the angular momenta of simple-ring systems—an account which will throw further light on the nature of Balmer's formula, and at the same time indicate the existence of certain primary elements not yet dealt with in former papers. Hitherto we have introduced only two—nebulium and protofluorine—and shown that in the case of protofluorine with n electrons, the angular momentum E of the atom rigorously follows the law

$$\frac{\mathrm{E}}{n} = \mathrm{A} + \mathrm{B}n + \mathrm{C}n^2,$$

where A, B, C are constants. In passing to an atom of another substance, they must be regarded as no longer constants, but functions of ν, the strength of the corresponding nucleus. A law expressed by the preceding formula has been described as harmonic —thus E/n is a harmonic function of n. It is natural to suppose that for a given number n of electrons and a variable nucleus νe, E/ν should be a harmonic function of ν; we shall work out the consequences of this supposition, which is found to be in accordance with astrophysical spectra. Thus we have, when $\nu = 5$,

$$\frac{\mathrm{E}}{n} = \mathrm{A} + \mathrm{B}n + \mathrm{C}n^2,$$

and when $\nu = n$, denoting some neutral atom,

$$\frac{\mathrm{E}}{\nu} = \alpha + \beta\nu + \gamma\nu^2.$$

The only formula which satisfies these requirements is, where A, B, . . ., are now constant,

$$\frac{\mathrm{E}}{n} = \mathrm{A} + \mathrm{B}\nu + \mathrm{C}\nu^2 - n(\mathrm{D} + \mathrm{E}\nu) + \mathrm{G}n^2$$

for an atom with n electrons and nucleus νe, n not being greater than ν. In practice, E is very closely equal to a multiple of Planck's unit in all cases, but there are slight divergencies which can only be expressed in terms of a subdivided unit, so that the coefficients A – G are not quite integral. But the smallness of G, for example, indicates that the divergencies have a simple explanation. It may be the case, for example, that account has to be taken of possible rotation of the nucleus of the atom after the manner of

a rigid body, at the expense of the angular momentum of the electrons, which would otherwise be *exactly* a multiple of $h/2\pi$. In any case, the rigorous character of the law, and the smallness of the deviations from exact multiples, indicate that it is valid, and that the terms in n^2 and ν^2 have some simple explanation of the kind suggested.

Without troubling further, therefore, about the exact relation of the angular momentum to Planck's h for the present, we shall choose, as a unit for E, the value used in an earlier paper, so that if A is the principal wave-length of a ring atom, E is given on a certain scale by

$$\frac{E}{n} = \lambda^{\frac{1}{3}}(\nu - \tfrac{1}{4}S_n)^{\frac{2}{3}}/(0{\cdot}062358).$$

We then have, for the various protofluorine systems:

$$\nu = 5, \quad n = 5, \quad \frac{E}{n} = 600$$

$$\nu = 5, \quad n = 4, \quad \frac{E}{n} = 658\tfrac{1}{2}$$

$$\nu = 5, \quad n = 3, \quad \frac{E}{n} = 718$$

$$\nu = 5, \quad n = 2, \quad \frac{E}{n} = 778\tfrac{1}{2}$$

$$\nu = 5, \quad n = 1, \quad \frac{E}{n} = 840.$$

The unit is that of Planck divided by $\lfloor\underline{5}$, and the values of $\frac{E}{n}$ are almost exactly in the ratios

$$10 : 11 : 12 : 13 : 14,$$

and their second differences, representing the small divergencies from these ratios, are the same throughout.

Inserting these values in the general formula for $\nu = 5$, $n = 1, 2, 3, 4, 5$, we obtain

$$G = \tfrac{1}{2}, \quad D + 5E = 63, \quad A + 5B + 25C = 902\tfrac{1}{2},$$

and no other independent equations.

Passing now to nebulium, whose neutral atom has a principal wave-length $\lambda = 5006{\cdot}89$, we find

$$\frac{E}{n} = (5006{\cdot}89)^{\frac{1}{3}}(3{\cdot}4226)^{\frac{2}{3}}/(0{\cdot}062358) = 576{\cdot}08.$$

Thus $$A + 4B + 16C - 4(D + 4E) = 568.$$

The weakness of the nebular spectrum makes uncertain the identifications of the lines of nebulium in any *individual* case treated in the paper on its spectrum. The only case to which a more absolute degree of probability may be attached is that of the

neutral atom just quoted, which gives the strong lines λ 5007 and λ 4363·5. In the case of the solar corona, a very definite and complete scheme of lines has been found with remarkable mutual relations, which eliminate the element of chance coincidence possible to nebulium as already dealt with. Accordingly, in calculating constants above, we have relied only on the cases of apparent certainty and derived four equations.

One identification for a nebulium system at least is definitely wrong,—that involving the line λ3869. This is shown by the fact that the correction for the mass of the nucleus, indicated in a later paper, while supporting the identifications for the neutral atom, makes the discrepancy between observed and calculated wave-lengths greater for this line. Apart from this conclusive reason, the strength of the line in nebulæ, and especially in Nova Persei at its nebular stage, could not be accounted for by the origin suggested.

Another line of reasoning, based on the discovery of the evolution products of the substance giving this line, which cannot be dealt with in this paper, has indicated its true origin, which has been rendered much more certain by the considerations below. It appears that it is a line of the neutral atom with nucleus $2e$.

The ratio of the principal lines for such an atom is

$$\left(\frac{2}{2-\frac{1}{4}}\right)^{\frac{1}{2}}\left(1+\frac{m}{M}\right)=1{\cdot}06905\left(1+\frac{m}{M}\right).$$

As a first approximation, we look for two lines in the nebular spectrum with a ratio 1·06905, and taking the most complete nebular spectra available—those of G.C. 4390 and Orion,—the only good pair is λ 4143 and λ 3869, whose ratio is 1·0708. If this is the required pair, we must have

$$1+\frac{m}{M}=(1{\cdot}0708)/(1{\cdot}06905)=1{\cdot}00166.$$

Now, the value of m/M for a nebulium atom, determined from the accurately measured lines of the neutral atom, is 0·000415, and, as stated in an earlier paper, the atomic weights of nebulium and protofluorine are found to be 1·3 and 2, with the help of the experimental value of m/M for a hydrogen atom. Since

$$\frac{1{\cdot}3}{2}=0{\cdot}65,$$

and

$$\frac{4^2}{5^2}=0{\cdot}64,$$

we may conclude that *in simple-ring systems, the inertia of the nucleus is proportional to the square of its charge.* This gives important information, of use in other connections, concerning the nature of the nucleus.

Since $4^2/2^2=4$, the value of m/M for a nucleus of charge $2e$

should be $4(0{\cdot}000415) = 0{\cdot}00166$, which is exactly the value above. Thus the correspondence of the observed wave-lengths with the calculated ones is apparently exact, and we may say on this ground alone that λ 4143 and λ 3869 are the lines of this neutral system, with some degree of certainty.

On this view, the fact that λ 3869 is often stronger than λ 5007, especially in the late stages of Nova Persei, receives an immediate explanation, for, as a line of a neutral atom, it should be comparable with the lines of neutral nebulium if the nova, or a part of it, has really been reduced to the simplest form.

Consider now the angular momentum per electron of this system. It is given by

$$\frac{E}{n} = (4143)^{\frac{1}{3}}(2 - \tfrac{1}{4})^{\frac{2}{3}}/(0{\cdot}062358) = 374{\cdot}04,$$

or 374 with great accuracy, of one part in 9000.

If neutral atoms follow a harmonic law, we can now calculate the wave-length of the line given by protohydrogen, and for this purpose we use the utmost accuracy available to test whether the law is rigid. Thus we take the principal line of protofluorine as 3987·1—it has been measured more frequently than others and is more accurate; of nebulium as 5007·9 (Wright); and of the substance with two electrons as 4142·9 (corresponding to 3868·9, the true wave-length of the line λ 3869 (Wright)); and find the sequence

	$\nu = 5$,	$\nu = 4$,	$\nu = 3$,	$\nu = 2$,	$\nu = 1$.
$\frac{E}{n} =$	600·00	576·08	...	374·04	...

These three members determine the harmonic sequence to be

600·00	576·08	500·76	374·04	195·92	
23·92	75·32	126·72	178·12		
51·40	51·40	51·40			

The second differences are exhibited also, and are exactly equal. For the line of protohydrogen, we therefore obtain a wave-length

$$\lambda = (195{\cdot}92 \times 0{\cdot}062358)^3 = 1823{\cdot}55.$$

Now, $1823{\cdot}55 = \frac{1}{2}(3647{\cdot}10)$, and *the limit of the hydrogen series is* 3647·14. Moreover, if the Balmer series is actually derived after the manner of the Wolf-Rayet formula, it should be of the form

$$\lambda = \frac{2\lambda_0}{1 + nd} \cdot \frac{m^2}{m^2 - 4},$$

where λ_0 is the wave-length of the line of protohydrogen. Evidently, therefore, $n = 0$, and the agreement is extraordinarily close. We may say therefore with confidence that *hydrogen is the immediate result, or the first evolution product, of protohydrogen*, and its early appearance in nebulæ is explained.

At the same time, the rigorous character of the harmonic law, which is evidently true to one part in 90,000, has linked up the spectra of hydrogen, protofluorine, and nebulium exactly in the same way as the protofluorine spectrum is linked up in itself, although nebulium and protofluorine are never found together in the same astrophysical spectrum.

It is interesting to determine the further evolution products of protohydrogen, but this must be reserved for another paper. Meanwhile, we shall briefly indicate one more simple-ring system which appears in the nebulæ. The value of E/n for the system with a nucleus $3e$, when neutral, is, as above, 500·76. This gives a principal wave-length

$$\lambda = (500{\cdot}76 + 0{\cdot}062358)^3 / \left(3 - \frac{1}{\sqrt{3}}\right)^2 = 5187.$$

Now, λ 5185 is a well-known nebular line, not due to hydrogen, helium, or nebulium. It is never strong, and has not been measured accurately. It disappears in older nebulæ like Orion, where the hydrogen and helium series, products of evolution, have become strong, but in the nebula G.C. 4390, which will be shown later to be a very primitive nebula, it was discovered first by Vogel as early as 1871. Its companion line for the neutral atom should have a wave-length

$$\lambda = 5187 \left(\frac{3 - \frac{1}{\sqrt{3}}}{3}\right)^{\frac{1}{2}} \left(1 + \frac{3}{2}\frac{m}{M}\right)^{-1} = 4657.$$

We cannot expect this to be visible in nebulæ, in view of the faintness of λ 5185. It is not the nebular line λ 4663, which has another origin. But it appears to be well known in the nebular stages of novæ, especially Nova Persei, where the maximum of the corresponding band is registered by various observers between λ 4650–4660. λ 5185 is also known in novæ.

A consideration of simple-ring systems with a nucleus of greater strength than $5e$ is postponed for another paper, where it will be shown, from a complete scheme of the angular momentum relations of the atoms, that there are exactly nine simple-ring systems possible, with the inclusion of protohydrogen—a fact which may have bearing on the periodic table of the terrestrial elements, and the recurrence of chemical properties in groups of eight elements.

On a New Algol-Type Variable Star in Pegasus. R.A. = 22^h 49^m 33^s, Decl. = +32° 27′·4 (1855·0). R.A. = 22^h 51^m 40^s, Decl. = +32° 41′·7 (1900·0). By A. Stanley Williams.

The approximate position given above is derived from measures of a photograph taken with a 4·4-inch portrait lens. The little chart, fig. 1, will help to identify the variable and the neighbouring stars.

The star is shown faint, nearly three-quarters of a magnitude below normal brightness, on a photograph taken with the above-mentioned portrait lens on 1901 December 10 (exp. 10^h 35^m to 11^h 34^m G.M.T.). Visual observations were commenced on 1904

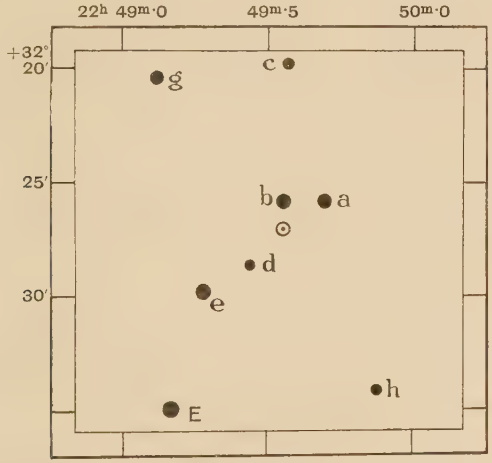


Fig. 1.

June 3, with the object of determining the period of what was evidently an Algol-type variable. In view of what follows, it is somewhat remarkable that at this, the first observation, the star should have been caught below normal brightness. Only two observations, separated by an interval of 20^m, could be secured before increasing daylight stopped work, but the star appeared to be increasing in brightness.

During the succeeding two years the variable was observed no less than 192 times upon 126 different nights without once being seen below normal brightness. At length on 1906 September 24 it was again seen faint. The observations were much interrupted by cloud, but a fortunate view obtained nearly an hour after the earlier ones showed that the star had then regained its normal brightness, and that the earlier observations must have been made during increasing light.

Finally, on 1907 August 27 the star was again seen faint. The

observations extend from a little before minimum brightness to about an hour after, and give a fair indication of the form of the light curve. (See fig. 2.)

The reason why the star should have been seen so seldom below normal brightness is now obvious. The whole period of light change is only about $2^h\ 30^m$, and only during two hours of this is the star at all conspicuously below normal brightness, whilst the interval between two successive minima is very nearly five and a quarter days. Consequently it is only for two hours at intervals of twenty-one days that a single observer is likely to have the opportunity of seeing the star faint, and such an observer might indeed very well watch the star for several years pretty closely without once observing it during the two hours when it is at all conspicuously faint.

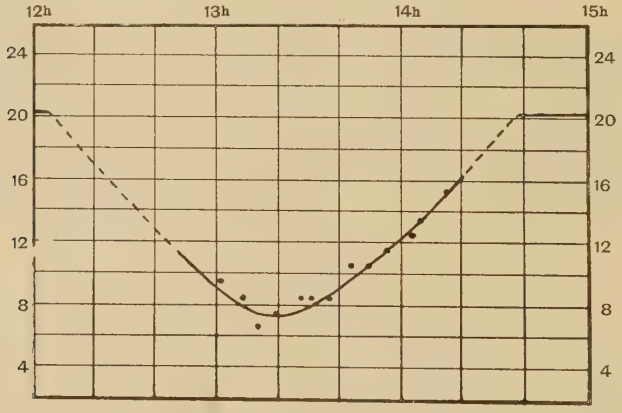


Fig. 2.

The approximate positions of the neighbouring stars shown in the chart, fig. 1, for the equinox 1855·0 will be found in the following table:—

Star.	R.A. h m s	Decl. ° ′	Steps.	Mag.
g	22 49 07·4	+32 20·6	...	...
E	22 49 10·5	+32 34·9	...	...
e	22 49 16·7	+32 29·7	...	...
d	22 49 25·8	+32 28·8	5	10·67
var.	22 49 33·5	+32 27·2	...	...
b	22 49 33·6	+32 26·1	18	10·09
c	22 49 34·0	+32 19·9	...	...
a	22 49 43·4	+32 26·0	20	10·00
h	22 49 52·4	+32 34·1	...	...

The star E is B.D. $+32^\circ 4551$, 9·4 mag. Column 4 of the

table gives the brightness of the comparison stars *a*, *b*, and *d* in steps, and column 5 the corresponding magnitudes on the assumptions that the star *a* is 10·0 mag. and the value of a step is 0·045 mag. The scale of steps has been formed chiefly from the observations made when the variable was faint, and it is necessarily rough, owing to the small number of observations, but it will be sufficient for present purposes.

On 1907 August 27 the variable was fortunately under observation for $1^h\ 13^m$, and the observations begin a little before the epoch of minimum, so that we can draw a fairly satisfactory representation of the light curve of the star during the period of change, assuming that the curve during the decrease in brightness is similar to that during the increase. This is shown in fig. 2. By means of this light curve the times of minimum on the nights of 1904 June 3, 1906 September 24, and 1907 August 27 could be derived with a satisfactory degree of exactness, as will be seen from the following little table, in which the first column gives the number of periods elapsed since the adopted epoch, column 3 the observed, and column 4 the computed times of minimum according to the concluded elements of variation, and the last column the differences O – C. The first observation is that of the photograph of 1901 December 10, and here it is only possible to say that the star must have been near minimum brightness at the time the photograph was taken.

E.	Date.	Obs. Min.*	C.	O – C.
0	1901 Dec. 10	J.D. 2415729·458	·460	– 0·002
172	1904 June 3	6635·542	·543	– ·001
332	1906 Sept. 24	7478·409	·410	– ·001
396	1907 Aug. 27	7815·556	·556	·000

The concluded elements of variation are :

Minimum = 1901 Dec. 10, $11^h\ 02^m$ (G M.T.) $+5^d\ 6^h\ 25^m\ 48^s$ E
= J.D. 2415729·460 $+5^d$·26792 E

The residuals in the last column are quite satisfactorily small, and the observations made during normal brightness are so numerous and so distributed as to preclude the possibility of any sub-multiple of the adopted period. The normal brightness of the variable is 20·2 steps = 9·99 mag., and the minimum brightness 7·5 steps = 10·56 mag., the range being thus 12·7 steps = 0·57 mag. The observations are given in detail below. The last two of those dated 1904 October 1, were made by Mr. P. S. Yendell, at Dorchester, Mass., U.S.A., with a $6\frac{1}{4}$-inch refractor, power 90. The brightness of the variable is given in steps, the addition of a colon signifying that the sky was very poor. In the last column will be found the phase in days and fractions of a day from the last preceding minimum.

* The times are not reduced to the sun.

1904.	G.M.T. h m	Observations.	Brightness.	Phase. d
June 3	13 46	a 10 v, b 5 v	15	0·032
,,	14 06	a 8 v, b 3 v	16	0·045
17	12 40	a 0 v 7 b 12 d	20	3·450
,,	13 42	About normal. Thin cloud.	20±	3·493
18	12 30	a 1 v 7 b	20	4·443
,,	12 56	Normal.	20±	4·461
20	12 15	a 0 v 3 b	20	1·164
,,	13 20	a 0 v 5 b	20	1·209
21	12 20	a 1 v 3 b	19·5	2·168
,,	13 14	a 0 v 4 b	20	2·205
22	12 30	a 0 v 3 b	20	3·175
,,	13 18	No change.	20±	3·208
23	12 25	a 0 v 5 b	20	4·171
,,	13 20	Normal.	20±	4·210
July 21	11 32	a 2 v 1 b	18·5	0·528
,,	12 05	No change.	20±	0·550
24	11 07	a 1 v 3 b	19·5	3·510
,,	12 29	a 0 v	20	3·567
26	13 30	Normal. Thin cloud.	20±	0·341
28	10 10	a 0 v	20	2·203
,,	11 10	Normal.	20±	2·244
31	10 50	About normal. Much cloud.	20±	5·210
Aug. 2	11 18	v=a	20	1·982
,,	12 50	v=a	20	2·046
,,	13 57	a 0 v 3 b	20	2·092
3	12 38	a 0 v 3 b	20	3·037
,,	14 09	a 0 v 3 b	20	3·101
8	9 55	a 1 v 4 b	20	2·656
,,	11 26	a 0 v	20	2·719
,,	12 30	a 0 v	20	2·764
,,	14 48	v 3 b	20	2·860
11	9 14	a 2 v 0 b	18	0·360
,,	9 33	a 3 v 0 b	18	0·373
,,	10 32	a 3 v 0 b	18	0·414
,,	12 31	a 3 v 0 b	18	0·497
,,	14 18	a 2 v 0 b	18	0·571
12	9 27	a 1 v 1 b	19	1·369
,,	10 46	a 1 v	19	1·424

1904.	G.M.T. h m	Observations.	Brightness.	Phase. d
Aug. 12	13 00	a 2 v	18	1·517
,,	14 30	a 1 v	19	1·579
13	9 49	a 2 v	18	2·384
15	9 40	a o v	20	4·378
,,	11 34	a o v	20	4·457
,,	14 18	a o v 4 b	20	4·571
17	9 23	a 1 v 2 b	19	1·098
,,	10 12	a o v	20	1·132
,,	14 30	a o v	20	1·311
18	9 00	a o v	20	2·082
,,	11 31	a o v	20	2·187
,,	15 16	a o v	20	2·343
19	10 23	a o v	20	3·140
,,	14 50	a o v	20	3·325
20	9 42	About normal.	20±	4·111
,,	11 48	a o v	20 :	4·199
,,	15 25	a o v	20	4·349
21	10 02	About normal. Uncertain. Cloud.	20±	5·125
24	10 18	a o v	20	2·868
,,	12 08	a o b	20	2·945
,,	14 20	a o b	20	3·036
27	9 50	Normal.	20±	0·581
,,	13 15	About normal. Cloud.	20±	0·723
28	8 17	a o v	20	1·516
,,	11 02	a o v	20	1·631
,,	15 43	a o v	20	1·784
29	9 03	a 1 v	19	2·548
,,	11 07	a 1 v	19	2·634
30	14 32	v 2 b	20	3·777
Sept. 2	9 11	a 2 v	18	1·286
3	9 25	a 2 v	18	2·295
,,	11 03	v 3 b	20	2·363
,,	14 50	a o v 3 b	20	2·715
4	9 58	About normal. Cloud.	20±	3·318
,,	12 46	a o v	20 :	3·435
5	8 50	a o v	20	4·271
,,	11 25	a 1 v	19	4·379
,,	14 45	a o v 4 c	20	4·518
7	9 12	a o v	20	1·018

1904.	G.M.T. h m	Observations.	Brightness.	Phase. d
Sept. 7	11 10	a o v	20	1·100
9	9 15	a 1 v	19	3·013
,,	11 04	a o v	20	3·096
10	9 28	a o v	20	4·029
,,	11 32	a o v	20	4·116
,,	12 30	a o v	20	4·156
11	8 34	a o v	20	4·972
,,	10 48	a 1 v	19 :	5·085
12	9 10	a o v	20 :	0·749
13	14 45	a o v	20 :	1·982
14	11 44	a 2 v	18	2·856
15	9 15	a 1 v	19 :	3·752
,,	12 40	a o v	20	3·895
16	8 55	a 1 v	19	4·739
,,	10 50	a 1 v	19	4·818
,,	13 57	a o v	20	4·948
17	9 12	a 1 v	19	0·482
,,	12 13	a 1 v	19	0·608
,,	15 28	a 1 v	19	0·743
18	8 41	a o v	20	1·461
,,	10 37	a o v	20	1·541
,,	15 36	a o v	20	1·749
19	9 10	a o v	20	2·481
,,	13 43	a 1 v	19	2·671
20	9 12	a 1 v	19	3·482
,,	13 42	a 1 v	19	3·670
21	9 20	a o v	20	4·488
,,	13 50	a o v	20	4·675
24	9 35	About normal.	20±	2·231
25	10 00	a o v	20	3·249
26	8 53	a o v	20	4·202
27	9 00	a 1 v	19	5·207
29	9 13	a o v	20	1·948
Oct. 1	9 16	a o v	20	3·950
,,	13 30	c 4 v, v 2 a (Yendell)	22	4·126
,,	13 53	v 1–2 a (Yendell)	22	4·142
3	8 44	a o v	20	0·660
,,	12 45	a 1 v	19	0·827
6	11 50	a o v	20	3·789
7	9 14	a o v	20	4·681

1904.	G.M.T. h m	Observations.	Brightness.	Phase. d
Oct. 8	9 35	a o v	20	0·427
12	9 28	a o v	20	4·422
13	9 30	a o v	20	0·156
14	9 28	a o v	20	1·154
18	9 22	a 2 v	18	5·150
,,	10 35	a 1 v	19	5·201
28	9 12	a o v	20	4·607
29	10 16	a o v	20	0·384
Nov. 3	9 25	a o v	20	0·080
5	7 12	a o v	20	1·988
12	6 49	a o v	20	3·704
15	8 54	a 1 v	19	1·523
Dec. 18	6 40	a 1 v	19	2·823
1905.				
Jan. 7	6 17	a 1 v	19	1·735
June 22	13 09	Normal.	20±	4·716
26	13 03	Normal.	20±	3·444
July 27	11 00	v=b	18	2·750
30	11 10	v 1 a	21	0·489
Aug. 1	12 43	v 2 a	22	2·554
4	10 38	a o v	20	0·199
5	12 07	v 1 b	19	1·261
,,	14 57	v 2 a	22	1·379
10	13 09	v 2 a	22	1·036
11	14 28	v 2 a	22	2·091
12	14 52	v 2 a	22	3·107
14	12 52	About normal.	20±	5·024
18	15 08	v 1 a	21	3·851
20	14 35	v 2 a	22	0·561
23	10 00	v=a	20	3·370
,,	14 51	v 1 a	21	3·572
29	10 21	v 2 a	22	4·116
,,	15 06	v 2 a	22	4·314
31	9 51	v 1 b	19	0·827
Sept. 3	11 04	v 2 a	22	3·878
7	10 09	v o ± a	20±	2·572
11	11 41	About normal.	20±	1·368
12	10 27	v 2 a	22	2·316
16	10 53	v 3 a	23	1·046
27	13 35	v 2 a	22	1·643

	G.M.T.	Observations.	Brightness.	Phase.
1905.	h m			d
Oct. 4	9 23	v 2 a	22	3·200
5	9 22	v 1 a	21	4·199
6	9 46	v 2 a	22	5·216
16	9 27	v 1 a	21	4·667
22	8 13	v 2 a	22	0·800
25	11 05	v 1 a	21	3·200
27	10 10	v 2 a	22	5·162
29	10 28	v 1 a	21	1·906
Nov. 7	9 35	v 2 a	22	0·333
16	9 25	v 3 a	23	4·058
20	9 33	v 2 a	22	2·796
23	8 47	v 1 a	21	0·496
27	9 28	v 1 a	21	4·524
1906.				
Jan. 6	6 25	v 0 a	20	2·254
July 24	11 53	v 2 a	22	1·301
26	12 20	v 2 a	22	3·320
27	12 25	a 2 v	18	4·323
Aug. 16	10 57	v 2 a	22	3·190
18	10 54	v 2 a	22	5·188
19	10 57	a 2 v	18	0·922
21	12 32	v 2 a	22	2·988
22	10 06	v 1 a	21	3·887
25	10 31	v 1 a	21	1·636
26	10 24	v 1 a	21	2·631
27	10 27	v 0 a	20	3·633
28	9 38	v 1 a	21 :	4·599
,,	14 17	v 1 a	21	4·793
29	9 29	v 1 a	21	0·325
30	9 28	v 1 a	21	1·324
,,	14 23	v 0 a	20	1·529
31	11 05	v 0 a	20	2·392
Sept. 6	9 37	v 2 a	22	3·063
9	10 18	v 2 a	22	0·823
11	10 46	v 2 a	22	2·843
16	10 35	v 2 a	22	2·567
19	10 49	v 1 a	21	0·310
20	10 44	v 1 a	21	1·306
22	10 45	a 1 v	19	3·307
24	10 09	a 6 v	[14]	0·014

1906.	G.M.T. h m	Observations.	Brightness.	Phase. d
Sept. 24	10 11	a 8 v 8 d	12·5	0·015
,,	10 17	a 8 v 8 d	12·5	0·019
,,	10 31	a 6 v 8 d	14	0·029
,,	10 34	a 7 v 7 d	12·5	0·031
,,	10 50	a 6 v 8 d	14	0·042
,,	11 42	a 0 v 14 d	20	0·078
25	8 15	a 0 v 18 d	20	0·935
,,	8 35	v 1 a	21	0·949
,,	9 07	v 1 a	21	0·971
,,	9 25	v 0 a	20	0·983
,,	9 35	v 1 a	21	0·990
,,	9 49	v 1 a	21	1·000
,,	10 25	v 0 a	20	1·025
,,	11 20	v 0 a	20	1·063
,,	12 17	v 1 a	21	1·103
,,	13 19	v 0 a	20	1·146
26	8 25	v 0 a	20	1·942
,,	9 08	v 1 a	21	1·972
,,	9 59	v 0 a	20	2·007
,,	12 10	v 0 a	20	2·098
,,	14 05	v 0 a	20	2·178
27	8 18	v 1 a	21	2·937
,,	9 15	v 0 a	20	2·976
,,	10 39	v 0 a	20	3·035
,,	11 11	v 0 a	20	3·057
Oct. 6	12 50	v 1 a	21	1·590
10	9 33	a 1 v	19	0·185
11	9 10	v 1 a	21	1·169
13	10 03	a 0 v	20	3·206
,,	13 38	v 1 a	21	3·355
14	7 55	a 0 v	20	4·117
,,	8 52	a 0 v	20	4·156
23	10 08	v 1 a	21	2·673
27	9 22	v 1 a	21	1·373
Nov. 11	7 25	a 4 v 7 d	[14]	0·488
,,	7 34	a 8 v 7 d	[12]	0·494
,,	7 49	a 6 v 6 d	[12·5]	0·505
,,	8 15	a 3 v 8 d	[15]	0·523
15	8 44	v 1 a	21	4·543
17	9 09	v 0 a	20	1·292

	G.M.T.	Observations.	Brightness.	Phase.
1906.	h m			d
Nov. 19	9 15	v 1 a	21	3·296
Dec. 7	8 14	v 0 a	20	0·183
1907.				
Aug. 27	13 02	a 9 v 3 d	9·5	5·255
,,	13 08	a 10 v 2 d	8·5	5·259
,,	13 13	a 12 v 0 d	6·5	5·263
,,	13 19	a 11 v 1 d	7·5	5·267
,,	13 27	a 10 v 2 d	8·5	0·004
,,	13 30	a 10 v 2 d	8·5	0·007
,,	13 36	a 10 v 2 d	8·5	0·011
,,	13 44	a 8 v 4 d	10·5	0·016
,,	13 49	a 8 v 4 d	10·5	0·020
,,	13 55	a 7 v 5 d	11·5	0·024
,,	14 03	a 6 v 6 d	12·5	0·029
,,	14 07	a 5 v 7 d	13·5	0·032
,,	14 15	a 3 v 9 d	15·5	0·038
28	9 30	a 0 v 12 d	20 :	0·840
,,	11 45	v 1 a	21 :	0·934
29	8 55	a 0 v 12 d	20	1·816
,,	9 24	v 1 a, v 13 d	21	1·836
,,	10 05	v 1 a, v 13 d	21	1·864
,,	11 01	v 2 a, v 14 d	22	1·903
Sept. 7	13 40	v 2 a, v 13 d	22	0·477
,,	14 34	v 1 a, v 13 d	21	0·515
8	11 40	v 1 a, v 13 d	21	1·394
,,	12 30	v 1 a	21	1·429
9	8 50	v 1 a, v 11 d	21	2·276
,,	9 23	v 1 a, v 12 d	21	2·299
,,	9 59	v 0 a, v 11 d	20	2·324
,,	12 37	v 1 a, v 11 d	21	2·434
10	9 03	v 1 a, v 12 d	21	3·285
,,	9 40	v 1 a, v 12 d	21	3·311
,,	13 04	v 0 a, v 12 d	20	3·452
11	8 51	v 1 a, v 13 d	21	4·277
,,	12 48	v 1 a, v 13±d	21	4·441

As already mentioned, the observations are so numerous and so distributed as to preclude the possibility of a sub-multiple of the adopted period. Also none of the observations making the star of normal brightness fall in the short period of change. There are, however, four rather largely discordant observations on the night of 1906 November 11, which make the star unduly faint. Nearly

all the observations of this variable were made with a power of 73 on a 6½-inch reflector; but in the observations of that night another eyepiece magnifying 110 times was used, probably because the stars were faint and difficult to observe with the lower power, and the time of year is particularly liable to the presence of mist, fog, haze, or dampness generally.

Forming the observations into groups of five,* but omitting all those made under unfavourable conditions, we have the following 45 mean values of the brightness of the star during normal light:—

Phase. d	Brightness.	Phase. d	Brightness.	Phase. d	Brightness.
0·115	20·0	1·679	20·4	3·207	20·6
·250	20·6	·841	20·8	·324	20·2
·392	18·8	·950	20·4	·438	19·8
·488	20·2	2·042	20·0	·590	19·9
·551	19·9	·125	20·3	·775	20·0
·732	19·8	·225	20·2	·924	20·6
·892	19·6	·315	20·2	4·113	21·0
·979	20·8	·411	19·8	·185	20·2
1·040	21·0	·587	20·6	·307	20·2
·116	20·0	·687	20·0	·413	20·0
·191	20·0	·795	20·2	·506	20·4
·296	20·2	·893	19·8	·624	20·4
·368	20·8	3·010	20·2	·796	19·8
·469	19·6	·071	20·4	5·157	20·0
·552	19·8	·162	21·2	·195	20·8

The mean of the 45 values is 20·2 steps = 9·99 mag. There is no indication of a secondary minimum nearly midway between two principal minima.

Hove:
1914 *January* 5.

On the Short Period Variable XZ Cygni. By C. Martin and H. C. Plummer.

1. The variability of XZ Cygni was discovered in 1905 by Mme. L. Ceraski from an examination of plates taken by M. S. Blajko. It received the provisional number† 76·1905, and is identical with B.D. +56°·2257 and A.G.C., Helsingfors, 10573. The designation of the star in the Second Harvard Catalogue‡ is 193056. Sixty-one photographic observations have been secured by Mr. Martin with the 15-inch reflector of this observatory

* The last group is formed of three fresh observations combined with the last two of the preceding group.

† *A.N.*, 4028.

‡ *Harvard Annals*, lv.

during the past autumn. All the plates have been measured and reduced by him.

2. The method of reduction has already been sufficiently described in papers dealing with the variables SU and SW Draconis.* For the elements of the star we have the determination of S. Enebo,†

$$\text{J.D. } 2417201{\cdot}25417 + 0^{d}{\cdot}46659\ \text{E}, \qquad \text{G.M.T.}$$

But the Dunsink observations give a maximum at

$$\text{J.D. } 2420062{\cdot}358,$$

and hence, if we infer that 6132 periods have elapsed in the interval, we have for the value of the period

$$0^{d}{\cdot}466586 = 11^{h}\ 11^{m}\ 53^{s}{\cdot}03.$$

This value, which depends on the not altogether safe assumption that the epochs of visual and photographic maximum can be identified, is in complete accord with that of Enebo, and has been adopted in the reductions.

3. The magnitudes of the comparison stars have been based, as before, on two plates exposed under similar conditions both on the region of the variable and on the χ Persei cluster. The magnitude scale of the plates is deduced from the stars in the latter region, for which we adopt the determinations of Schwarzschild. The results are given in the following table:—

TABLE I.

Star.	B.D.	Plate 1177. m_1	Plate 1183. m_2	Diff. $m_2 - m_1$	Provisional Ph. mag.
a	+55°·2216	8·69	8·63	−·06	8·66
b	+55·2226	9·65	9·69	+·04	9·67
c	+55·2219	9·74	9·75	+·01	9·75
d	+56·2269	10·18	10·20	+·02	10·19
e	+56·2266	10·45	10·50	+·05	10·48

4. Each exposure on the variable gives a set of residuals for the comparison stars. From an examination of these the following results are obtained:—

Star	*a*	*b*	*c*	*d*	*e*
	m	m	m	m	m
Extreme range	·11	·19	·13	·13	·11
Probable error	±·020	±·030	±·021	±·019	±·019
Mean correction	+·010	−·029	−·004	−·028	+·029
Final magnitude	8·67	9·64	9·75	10·16	10·51

Here the extreme range is the sum of the largest positive and negative residuals in the series, each taken positively. The

* *M.N.*, lxxiii. pp. 166, 440.

† *Beob. Veränderlicher Sterne, angestellt auf Dombaas (Norwegen)*, ii.

probable error has been calculated in a summary way, and refers to the result from a single exposure. The mean corrections are the means for each series of residuals, and when applied to the provisional magnitudes of Table I. give the final magnitudes of the last line. These corrections are sensible, and have been allowed for by subtracting the mean corrected residual for each plate from the magnitude of the variable as based on the provisional magnitudes of the comparison stars. This plate correction, however, in no case exceeds $0^m{\cdot}02$.

5. The details of the observations of XZ Cygni are given in Table II. The second column gives the date and G.M.T. of the middle of the exposure. The next column gives the corresponding Julian date with decimal fraction of a day from G.M. noon. The phase in the fourth column is expressed as a decimal fraction of a day, and is reckoned from the preceding epoch of maximum, corresponding to J.D. 2420062·358 as given in §2. The time of exposure is given in the fifth column, and the concluded photographic magnitude of the variable in the sixth column. The penultimate column shows the deviations of the observations from an interpolated curve the derivation of which has still to be explained (§ 7). Finally, the quality of the images according to the impression formed in the course of measuring the plates is indicated in the last column. The reduction to heliocentric time has not been applied.

Table II.

Serial No.	Date G.M.T.	Julian Day.	Phase in days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1913. h m	2420		min.			
1	Aug. 25. 11 54	005·496	·062	35	9·53	+ ·02	Good.
2	26 10 4	006·420	·053	30	9·49	+ ·05	Good.
3	27 11 43	007·489	·189	30	9·98	− ·01	Good.
4	27 12 13	007·509	·209	30	10·05	− ·04	Good.
5	28 10 38	008·444	·210	30	10·07	− ·03	Fair.
6	28 11 9	008·464	·230	30	10·10	+ ·03	Fair.
7	28 11 49	008·492	·258	30	10·15	− ·03	Fair.
8	28 12 19	008·514	·280	30	10·10	− ·02	Fair.
9	Sept. 3 11 18	014·471	·172	35	9·94	− ·01	Good.
10	3 11 54	014·496	·197	35	10·04	+ ·01	Good.
11	9 10 24	020·434	·069	30	9·55	+ ·02	Fair.
12	9 10 59	020·458	·093	30	9·63	− ·02	Good.
13	9 11 38	020·485	·120	30	9·81	+ ·03	Fair.
14	9 12 9	020·506	·141	30	9·86	+ ·06	Fair.
15	30 9 28	041·395	·034	35	9·29	+ ·02	Fair.
16	30 10 4	041·420	·059	35	9·50	·00	Good.
17	Oct. 9 11 20	050·472	·245	30	10·07	− ·03	Good.
18	9 12 35	050·525	·298	30	10·12	− ·07	Fair.
19	20 7 47	061·325	·367	30	10·14	− ·03	Fair.
20	21 6 58	062·291	·400	30	10·26	− ·04	Fair.

TABLE II.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Phase in days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1913.	h	m	2420		min.			
21	Oct. 21	7	29	062·313	·422	30	10·05	+ ·06	Good.
22	21	8	7	062·337	·446	30	9·34	− ·02	Fair.
23	21	8	38	062·360	·002	30	8·94	+ ·03	Fair.
24	22	6	55	063·288	·463	30	8·93	+ ·01	Good.
25	22	7	32	063·314	·023	30	9·15	− ·04	Fair.
26	22	8	3	063·335	·044	30	9·31	− ·04	Fair.
27	22	8	40	063·361	·070	30	9·54	·00	Good.
28	23	7	1	064·293	·069	26	9·50	− ·04	Fair.
29	23	7	32	064·314	·090	30	9·66	+ ·05	Fair.
30	23	8	4	064·336	·112	30	9·73	− ·05	Fair.
31	23	8	44	064·365	·141	30	9·82	+ ·02	Good.
32	24	7	2	065·293	·135	30	9·77	− ·01	Fair.
33	24	7	34	065·316	·158	30	9·85	− ·05	Fair.
34	24	8	10	065·340	·182	30	10·02	+ ·06	Fair.
35	24	10	51	065·453	·295	40	10·19	+ ·01	Fair.
36	24	11	31	065·479	·321	40	10·17	+ ·02	Fair.
37	25	8	5	066·337	·246	40	10·16	+ ·05	Fair.
38	26	7	15	067·302	·278	40	10·11	− ·01	Fair.
39	26	7	56	067·331	·307	40	10·20	− ·01	Fair.
40	29	10	30	070·438	·148	30	9·94	+ ·10	Good.
41	29	12	18	070·513	·223	30	10·10	+ ·01	Good.
42	30	7	20	071·306	·083	30	9·55	− ·02	Good.
43	30	7	50	071·327	·104	30	9·67	− ·07	Good.
44	Nov. 2	9	14	074·385	·362	30	10·32	+ ·11	Bad.
45	2	9	44	074·407	·384	30	10·31	+ ·06	Fair.
46	2	10	18	074·430	·407	30	10·28	+ ·04	Fair.
47	2	10	49	074·451	·428	30	9·90	·00	Fair.
48	3	7	7	075·296	·340	30	10·36	+ ·09	Fair.
49	3	7	58	075·333	·377	30	10·20	+ ·03	Fair.
50	3	8	49	075·367	·411	30	10·20	+ ·02	Fair.
51	3	9	19	075·389	·433	30	9·71	− ·03	Fair.
52	3	9	50	075·410	·454	30	9·11	+ ·01	Fair.
53	3	10	29	075·438	·015	30	9·13	+ ·04	Fair.
54	3	11	3	075·460	·037	30	9·26	− ·04	Fair.
55	24	6	51	096·286	·334	30	10·24	+ ·03	Good
56	24	7	21	096·306	·354	30	10·32	+ ·03	Good.
57	26	7	10	098·299	·013	30	9·16	+ ·10	Fair.
58	26	7	40	098·320	·034	30	9·35	+ ·09	Fair.
59	Dec. 1	6	41	103·279	·328	30	10·14	− ·03	Fair.
60	1	7	1	103·293	·342	18	10·30	+ ·02	Fair.
61	1	9	20	103·389	·438	28	9·49	− ·04	Fair.

6. It will be noticed that there is an interval of a whole magnitude between the two comparison stars *a* and *b* in Table I. The reason for this is that an intermediate star, B.D. $+56^{\circ}\cdot2273$, proved to be itself variable. It is of interest to remark that, according to our experience of the stars selected as standards of comparison, about one star in five or six shows variation in its light of an amount which can be detected. The measured magnitudes of this star are given in Table III., together with the serial numbers which allow the corresponding dates and other relevant information to be taken from Table II. The mean photographic magnitude is 9·29, the observed limits being 8·98 and 9·50. The light of the star fluctuates rapidly within this range of half a magnitude, and the type of variation seems to be irregular or at any rate complex.

TABLE III.

No.	Mag.	No.	Mag.	No.	Mag.	No.	Mag.
1	8·98	17	9·49	33	9·46	49	9·48
2	9·11	18	9·33	34	9·21	50	9·31
3	8·99	19	9·20	35	9·44	51	9·31
4	9·14	20	9·12	36	9·29	52	9·33
5	9·21	21	9·30	37	9·26	53	9·27
6	9·20	22	9·29	38	9·21	54	9·24
7	9·22	23	9·24	39	9·29	55	9·32
8	9·22	24	9·21	40	9·03	56	9·50
9	9·31	25	9·40	41	9·32	57	9·22
10	9·23	26	9·30	42	9·29	58	9·18
11	9·30	27	9·30	43	9·39	59	9·25
12	9·48	28	9·23	44	9·31	60	9·27
13	9·47	29	9·28	45	9·28	61	9·34
14	9·43	30	9·39	46	9·26		
15	9·33	31	9·34	47	9·26		
16	9·39	32	9·48	48	9·24		

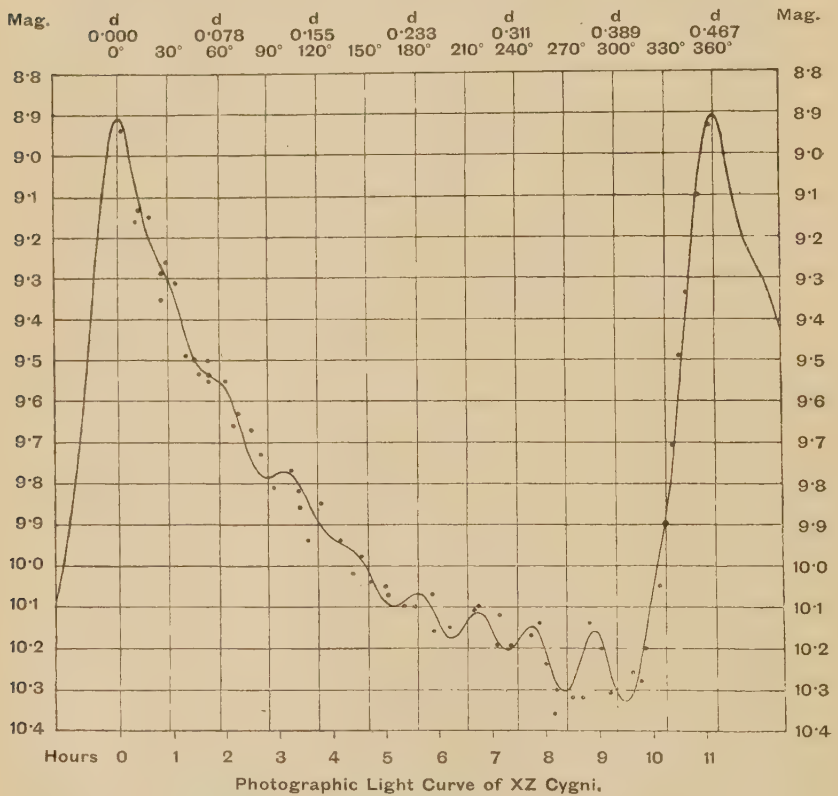
7. The observed magnitudes of XZ Cygni were plotted in the usual way, and a tolerably smooth curve drawn through the points. Twenty-four equidistant ordinates were then read off, beginning with the maximum, and these were then submitted to harmonic analysis.

The result is to represent the magnitude by the formula:

$$\begin{aligned} m = 9\cdot86 &+ \cdot44 \sin(\theta + 231^{\circ}) + \cdot23 \sin(2\theta + 239^{\circ}) + \cdot16 \sin(3\theta + 258^{\circ}) \\ &+ \cdot09 \sin(4\theta + 269^{\circ}) + \cdot07 \sin(5\theta + 290^{\circ}) + \cdot05 \sin(6\theta + 302^{\circ}) \\ &+ \cdot02 \sin(7\theta + 326^{\circ}) + \cdot01 \sin(8\theta + 52^{\circ}) + \cdot01 \sin(9\theta + 248^{\circ}) \\ &+ \cdot04 \sin(10\theta + 287^{\circ}) + \cdot02 \sin(11\theta + 346^{\circ}) \end{aligned}$$

where θ runs from the maximum and completes its period 360° in the period of the star. The considerable term with argument

10θ seems noteworthy. The curve in the figure corresponds to this formula, with the small terms depending on 8θ and 9θ omitted. This is the curve with which the observed magnitudes have been compared, the residuals being given in the seventh column of Table II. From these we should estimate the approximate probable error of a single observation of the variable at $\pm$ ^{m}029—a figure which, to judge by the corresponding figures for



Photographic Light Curve of XZ Cygni.

the comparison stars, suggests that in drawing the curve a fair balance has been held between conceding too much and too little to the individual observations.

8. In a recent note * the need for caution in interpreting the results of harmonic analysis was emphasised. Yet it is difficult to resist the temptation to make a comparison with results apparently similar. Let us then compare the phases of the terms found above with the phases of the series found † to represent Guthnick's visual light-curve of ST Virginis. Thus we have:

* *M.N.*, lxxiii. p. 652. † *Ibid.*, p. 654.

TABLE IV.

n.	ST Virginis (1).	XZ Cygni (x).	Diff. ($x-1$).
1	238°	231°	−7°
2	238	239	+1
3	258	258	0
4	269	269	0
5	286	290	+4
6	302	302	0
7	321	326	+5

The phases of the higher terms, which on the one side or the other are practically negligible, naturally differ widely. But that the resemblance extends so far must apparently be due to something more than chance, the more so as the stars belong to the half-dozen which alone of this type have been discussed by this method. On the other hand, for the amplitudes of the terms we find a closer analogy in S Arae, which furnishes the comparison (unit $^{m}\cdot01$):—

n.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
S Aræ . . .	47	24	16	10	7	5	4	3	2	1	1
XZ Cygni . . .	44	23	16	9	7	5	2	1	1	4	2

Here again the resemblance seems remarkably close. It is certainly a curious thing that, in the main at all events, we can reproduce the curve of XZ Cygni by combining the amplitudes of S Aræ with the phases of ST Virginis.

9. But if the harmonic representation of the curve reveals affinities with two stars, both of which led the visual observers to infer a stationary phase at minimum, the graphical representation, on the other hand, points to a remarkably close relationship with SW Draconis.* Thus we see in both stars the same peculiar double minimum, preceded by three or four premonitory fluctuations. These are features of which, standing alone, in the absence of very numerous observations, we might doubt the reality, and it is well that they should be confirmed by parallel instances. Nor is the similarity confined to the mere existence of these features: it extends even to their individual character. Thus the first minimum of SW Draconis is approached by a sudden drop, the abruptness of which overtaxes the power of a limited Fourier series to represent it. The same tendency is to be observed in XZ Cygni, and at approximately the same phase in the period (near $\theta = 255°$). In both curves, again, a sudden drop is suggested near the phase 115°. As a connecting link between different members of its type, XZ Cygni seems to be of special value. Its range of photographic magnitude is about 1·4. Mr. Enebo found a visual range of about 0·7, but in a letter for which we are greatly indebted to

* Cf. *M.N.*, lxxiii., Pl. 17 (facing p. 448).

him he states that later observations with a more powerful instrument have shown the range to be considerably greater than this.

10. Reference has been made to a recent note by one of us on variable stars of cluster type, and the opportunity may be taken to correct some impressions which were not intended but might not unfairly be drawn. The very difficult question of smoothing observations was alluded to in terms which might appear too dogmatic. As some justification it may be remarked that it is far easier for the eye to ignore those details of a curve which may appear irrelevant or ill-founded than to supply such as have been suppressed. Thus features which may ultimately prove characteristic are apt to be overlooked, and proper classification may be needlessly confused or retarded. But though it would not be difficult to illustrate the contention that the legitimate limits of the process of smoothing observations have been in some instances exceeded, the reference to Dr. Roberts' work on S Aræ in this connection is misleading. Dr. Roberts has done so much valuable work in investigating the accidental and systematic errors to which this class of observations is liable, that this point deserves a little explanation. It was simply urged that a continuous variation within very narrow limits was as consistent with the observations of S Aræ as the hypothesis of a stationary phase. But it was not intended to imply that the latter hypothesis was contradicted by the observations. As a matter of fact, our photographic observations have in no case revealed a stationary phase. The reason for this may be merely that we have only examined a few stars as yet. But there may be another reason. We know that the range of photographic magnitudes is in general greater than the visual range in this type of variation. This may be expressed as a magnification of the photographic light-curve. Now is there any reason to assume that this magnification is uniform at all parts of the curve? On the contrary, it may be that it is more pronounced in the region of the minimum than elsewhere. If so, the variations near this point of the visual curve may well be so small as to escape detection.

11. Incidentally, as the point is one of some importance, an example presents itself which may have some bearing upon it. The remarkable variable XX Cygni (period little more than 3^h) has been discussed very completely by Kron.* At the end of his memoir are given the light-curves derived from the work of a considerable number of observers. With one exception all the different series were made by visual observers. The exception is a photographic series by Parkhurst and Jordan.† The visual curves agree at least in showing a minimum of normal type. So indeed does the mean photographic curve as drawn. When, however, the diagram is examined, it is immediately apparent that the curve gives residuals to the mean observations which seem unexpectedly large.

* *Pub. des Astrophysik. Obs. zu Potsdam*, xxii. No. 65.
† *Ap. J.*, xxiii. p. 84.

It is, however, possible that the curve should have been drawn to follow the observations much more faithfully. If this were done, we should find a striking resemblance to the curves found to represent the variations of XZ Cygni and SW Draconis. In particular, we see the same abrupt drop preceding minimum, and it so happens that the position of this drop is at precisely the same point in the period, at the phase $\theta = 255^\circ$, or about five-sevenths of the period after maximum. Unfortunately it is not possible to insist quite firmly on this interpretation, which rests on a fairly literal acceptance of the normal points, because the individual observations are undeniably of indifferent quality, if indeed they are consistent with the hypothesis that the variation of the star is regular in the period assigned. But the explanation suggested would do something to rehabilitate the observations, and, in view of the period, would add an interesting member to what may prove a distinctive type. The larger fluctuations near minimum in the photographic light-curve are not altogether happily expressed as a magnification of the ordinates, if the photographic range is, as it appears to be, actually less than the visual in the case of XX Cygni.

12. Another point referred to Mr. Shapley's treatment of S Antliæ as an ellipsoidal star. Whether the curve assigned fitted the observations particularly well may be a matter of opinion. But it was pointed out that, according to all the signs, S Antliæ ought to be attributed to Bailey's sub-class *c* of the cluster variables. It should perhaps have been added that this classification is not necessarily inconsistent with the hypothesis of ellipsoidal form. Indeed, it would be very interesting if it were found that this theory could be applied to a whole group of stars. It would thus gain a measure of weight which it cannot have so long as it is applied only to a few instances selected from the great variety of curves available for discussion. There is a test to which this theory in its simplest form is subject, and which is easily applied. For the visual and photographic range of variation should agree fairly closely. A certain difference might be attributed to the effect of selective absorption towards the limb of the star.

As regards the vexed question whether the so-called cluster type forms a group apart from the δ Cephei type, any decision at the present time seems premature. In the meanwhile the soundest principle of classification is to draw the lines as broadly and simply as possible.

Dunsink Observatory:
1914 *January* 2.

The Nebula H.IV. 69 Tauri, N.G.C. 1514, and the Fainter Nebulæ, N.G.C. 821–2655–2782–2787–2830–2859–2880.

By Mrs Isaac Roberts. (Plates 8–10.)

(*Communicated by the Secretaries.*)

The nebula H.IV. 69 Tauri (*h* 311, N.G.C. 1514, G.C. 810), the first planetary nebula discovered, was described by Sir William Herschel in November 1790 as "a most singular phenomenon: a star 8th magnitude, with a faint luminous atmosphere of a circular form, about 3′ in diameter. The star is perfectly in the centre, and the atmosphere is so diluted, faint, and equal throughout, that there can be no surmise of its consisting of stars, nor can there be a doubt of the evident connection between the atmosphere and the star."†

At Birr Castle, between the years 1852 and 1873, twenty observations were made of the nebula; they were accompanied by a sketch inserted in the text, and an engraving fig. 7, pl. xxv., P.T. 1861. The nebula is successively described as follows:—

"1852. Jan. 13. New spiral of an annular form round the star, which is central. Brightest part is *sf* the star, spirality is very faint, but I have no doubt of its existence.

1855. Oct. 7. Annular, but with a break in the south side of the annulus. . . .

1858. Jan. 13. . . . the whole edge was ragged and irregular and the whole nebula much mottled. [See *Phil. Trans.*, 1861, pl. xxv. fig. 7.]

1862. Nov. 20. Very difficult to determine its true shape. . . .

1873. Nov. 16. Star 8th magnitude in a nebulous atmosphere which is irregular in brightness, being somewhat denser *sf* and somewhat fainter *np*. . . ."‡

The nebula H.IV. 69 Tauri was photographed at Starfield, Crowborough, in 1890, 1892, 1902, and is found in the Isaac Roberts Collection of Celestial Photographs on the following plates:—

I.R.* 1803. Dec. 13, 1890.
Exp.: $4^h\ 42^m$ to $5^h\ 28^m$ S.T. Crowborough ($0^h\ 46^m$)
1804. Jan. 1, 1892.
Exp.: 2 32 to 4 52 „ „ (2 20)
1822. Jan. 14, 1902.
Exp.: 3 33 to 5 03 „ „ (1 30)
3025. Jan. 30, 1902.
Exp.: 3 35 to 5 05 „ „ (1 30)

† Smyth and Chambers, *A Cycle of Celestial Objects*, p. 102.

‡ "Observations of Nebulæ and Clusters of Stars made with the 6-foot and 3-foot reflectors at Birr Castle from the year 1848 up to the year 1878, by the Right Hon. the Earl of Rosse, LL.D., D.C.L., F.R.S."

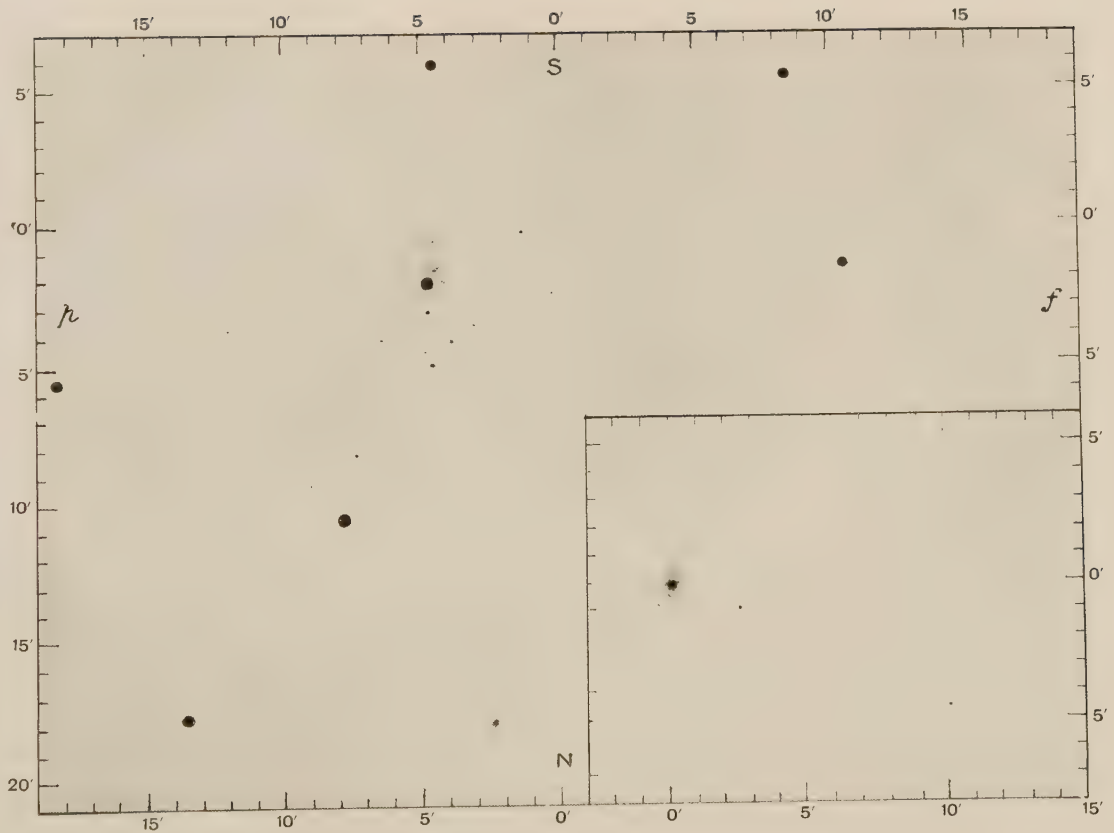
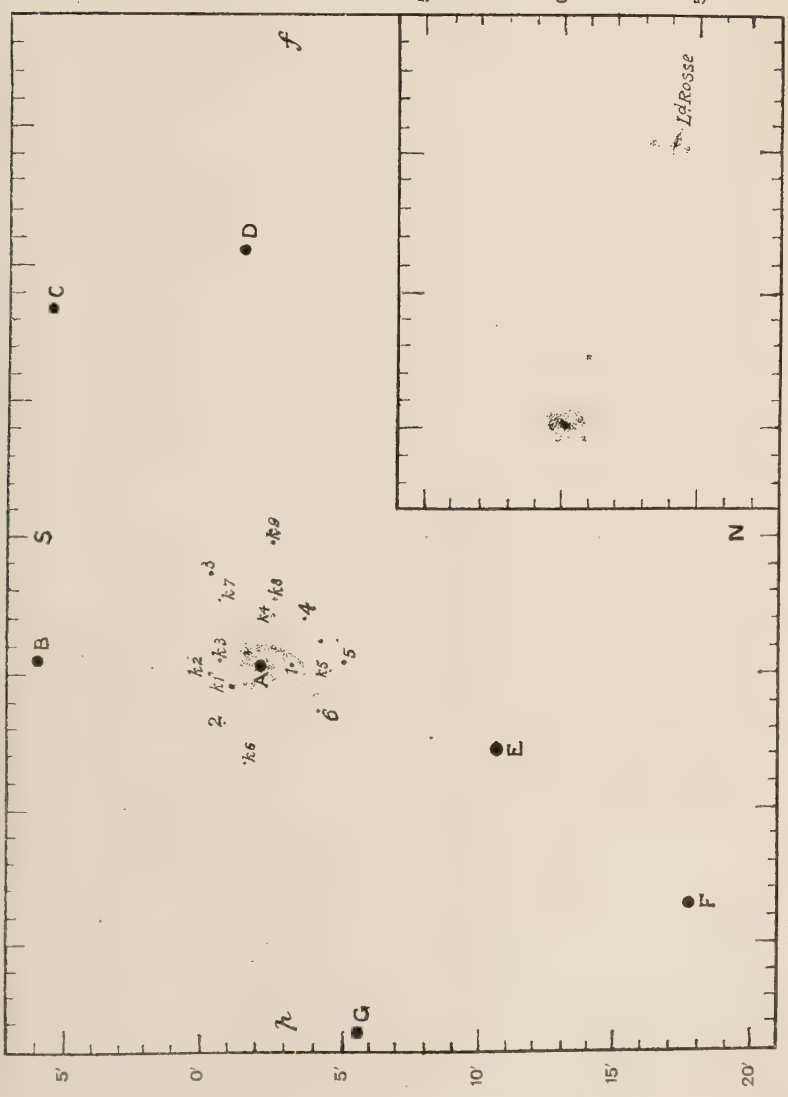


Fig. 1.—I.R. * 3025. H.IV. 69 Tauri.

Fig. 2.—I.R * 1895. H.I. 137 Lyncis.


S
N
f
h
A
B
C
D
E
F
G
k1
k2
k3
k4
k5
k6
k7
k8
k9
1
2
3
4
5
6
La Rosse
KEY MAP.

and on the 5-inch camera plates I.R.* 1823 and 1356 obtained simultaneously with the reflector plates 1822 and 3025.

Description.—The photograph I.R.* 3025 shows the nebula to be formed by a bright stellar nucleus (star B.D. +30° 623 mag. 8·8, Int. 2, diameters 31″ 25″) involved in nebulosity which seems to be of a spiral structure. In the immediate vicinity of the nucleus the nebulosity is extremely faint (Int. 18); large patches of denser nebulosity (Int. 14, 15, 16 . . .) irregular in shape, tapering toward the nucleus on its *sf* and *n* sides, are grouped along a broken annulus or one or two spiral arcs. The patches are indicated by the letters *ab*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, in the description on p. 237.†

The spiral structure of H.IV. 69 becomes more apparent when the image of the nebula is viewed with the *f* side of the plate downward; but it is difficult to determine from the plate I.R.* 3025, which is even denser and richer in detail than I.R.*1822, whether the nebula presents a single left-hand spiral arm—formed by the nebulous patches *f*, *g*, *h*, the scattered and very diffused patches *i*, the denser patches *ab*, *c*, the diffused patch *d*, and the widely scattered patches *e*—or whether the nebula consists of two spiral arms, the following arm being defined by the patches *ab*, *c*, the patch *d*, where the nebulosity decreases in intensity, broadening northward of star I and breaking up into fainter patches lined by a stellar curve; the preceding arm defined by the large, diffused and intermingling patches *f*, *g*, *h*, and the widely scattered patches *i*, among which are the nebulous knots or very faint stars *k*1, *k*2, *k*3.

The latter interpretation corresponds with the structure observed at Birr Castle in 1858, but the arms of the nebula, as represented on plate xxv. fig. 7, P.T. 1861, must be extended *n* and *s*. Fig. 7 is oriented with *p* above and *n* to the right.

The denser part of the nebula covers about 115″ from *f* to *p*, and about 106″ from *n* to *s*, and 180″ from star 1 to *k*2. The nebula seems to extend over 3′ from *p* to *f* and over 4′ from *n* to *s* when its extremely faint exterior nebulosity is taken into consideration.

The table (p. 236) contains the positions of the denser patches of nebulosity involved in the nebula H.IV. 69 Tauri, the positions of the nebulous knots or faint stars in its vicinity, and of the standard stars of the I.R.* plate 3025. The measures were made by me with the Isaac Roberts instrument (*M.N.*, 1888, pp. 5–13). The constants of the plate were determined by Mr. Pourteau of the Paris Observatory; the measures were reduced at By-Thomery with the assistance of Mdlle Madeleine Bougréau.

† On the I.R.* plates 1803, 1804 the nebulosity surrounding the stellar nucleus is barely visible. On the camera plates 1823–1356, H.IV. 69 appears like a dense star surrounded by extremely faint nebulosity little extended.

PLATE I.R.* 3025.

Places for 1900·0 *of the Standard Stars derived from the Astrographic Catalogue.*

	Oxford Zone. $3^h\ 58^m+31°$.			Oxford Zone. $4^h\ 7^m+31°$.			Oxford Zone. $4^h\ 3^m+30°$.		
	No.	R.A.	Decl.	No.	R.A.	Decl.	No.	R.A.	Decl.
		s	″		s	″		s	″
H	7751	29·84	2·4	...	...	...	7021	29·96	2·9
G	7752	57·26	0·5	...	...	...	7022	57·32	0·4
F	7791	18·20	10·1	8066	18·24	10·0	7058	18·30	10·6
E	7766	46·10	8·8	8058	46·11	8·5	7033	46·10	8·5
A	7753	0·16	38·1	8041	0·21	37·3	7023	0·24	38·7
B	7732	1·32	40·0	8033	1·39	39·6	7008	1·34	40·4
C	...	...	...	8034	1·63	7·3	7009	1·56	8·6
D	...	...	...	8043	11·55	3·5	7012	11·50	4·0

	Adopted Places from which the Constants were derived.		Measured Rectangular Co-ordinates.		Concluded Positions for 1900·0 of the Standard Stars.			Diameters of Star Images.		Mag.	B.D.
	s	″	′	′	h m s	°	′ ″	″	″		
H	29·90	2·7	−24·126	+ 3·680	4 1 29·85	30	32 2·2	33	26	9·0 K	+30° 618
G	57·29	0·5	−18·208	+ 5·583	1 57·27		34 0·4	25	21	9·2 K	+30 619
F	18·25	10·2	−13·587	+17·689	2 18·27		46 10·2	28	23	9·3 B	+30 620
E	46·10	8·6	− 7·657	+10·627	2 46·16		39 9·4	33	31	8·3 K	+30 621
A	0·20	38·0	− 4·706	+ 2·099	3 0·18		30 38·7	31	25	8·8 −	+30 623
B	1·35	40·0	− 4·506	− 5·866	3 1·37		22 40·4	30	29	8·5 K	+30 624
C	1·60	8·0	+ 8·478	− 5·510	4 1·63		23 7·2	28	27	8·6 K	+30 626
D	11·53	3·8	+10·635	+ 1·412	4 4 11·49	30	30 3·7	25	23	9·4 −	+30 627

The Constants and Concluded Centre of the Plate I.R. 3025.

$\tau x = +0''.00104$ $\tau y = +0''.00100$

$ix = -0''.00633$ $iy = -0''.00734$

$\alpha_0 = 4^h\ 3^m\ 22^s.11$ $\delta_0 = +30°\ 28'\ 34''.8$

The Nebula H.IV. 69 *Tauri.*

Object.	Measured Co-ordinates.		R.A. 1900·0.	Decl. 1900·0.	Diameters.		Intensity.
	′	′	h m s	° ′ ″	″	″	
N	−4·706	+2·099	4 3 0·18	30 30 38·7	31	25	...
h	−5·429	+1·930	2 56·82	30 28·2	7	7	14
g	−5·230	+2·592	2 57·73	31 8·1	9	6	14
ab	−4·431	+1·584	3 1·47	30 7·9	18	17	14
c	−4·089	+2·098	4 3 3·05	30 30 38·9	26	18	15

Small Stars in the Vicinity of H.IV. 69.

			h m s	° ′ ″	″	″	
K_6	−8′·231	+1′·467	4 2 43·82	30 29 58·9	12	9	17
*2	−6·844	+0·771	2 50·29	29 17·9	9	6	12
*6	−6·258	+4·086	2 52·90	32 37·2	9	8	12
K_1	−5·564	+1·033	2 56·23	29 34·2	11	8	16
K_2	−5·028	+0·264	2 58·74	28 48·3	9	7	15
K_5	−4·839	+4·634	2 59·48	33 10·9	9	7	16
*1	−4·737	+3·236	3 0 00	31 46·9	9	7	7
K_3	−4·536	+0·565	3 1·01	29 6·7	14	10	17
*5	−4·505	+4·966	3 1·02	33 31·0	12	7	6
*4	−3·025	+3·743	3 7·94	32 18·3	8	7	10
K_4	−2·873	+2·611	3 8·68	31 10·3	8	7	18
K_8	−2·264	+2·567	3 11·51	31 8·0	8	7	16
K_7	−2·232	+0·616	3 11·72	29 10·8	8	8	18
*3	−1·277	+0·293	3 16·17	*28 51·8	12	9	4
K_9	−0·173	+2·623	4 3 21·23	30 31 12·3	12	8	17

Definition and Description of the Objects measured.

N, stellar nucleus of H.IV. 69, slightly elongated *sp* to *nf*, as the stars of the same region of the I.R.* plate 3025.

ab, large nebulous patch in the *sf* region of the nucleus, like two condensations in line; the central part of *ab* was measured; very difficult.

c, large nebulous patch following the nucleus, like a group of nebulous knots involved in fainter nebulosity; whole mass measured; there is a break between the patches *ab* and *c*.

f, group of small nebulous knots in line, north of N; not measured.

g, nebulous patch in the *np* region of the nucleus, like a group of diffused nebulous knots forming the *np* arc; the densest knot was measured.

h, diffused patch of nebulosity with nebulous knots involved, of which the densest was measured.

i, group of very faint and scattered nebulous knots or faint stars among which *k*1, *k*2, *k*3 were measured.

*k*4, extremely faint nebulous knot involved in fainter nebulosity.

*k*5, faint nebulous knot or very faint star, in a curve of small stars and very faint nebulous knots encircling the nebula, as is shown by the photograph.

*k*6, *k*7, *k*8, as above.

*k*9, nebulous knot or very faint star.

*1, small star situated beyond the denser northern margin of the nebula; a faint arc of nebulosity directed *sf* to *np* passes over the star; there are numerous faint knots in the area limited by the stars 1, 4, 5, 6.

2, small star in the stellar curve encircling the nebula.
3, dense star near the centre of the plate I.R.* 3025.
4, 5, 6, see above. Two very faint nebulous knots or extremely faint stars are near the *n* margin of star 5.

The stars represented on fig. 7, pl. xxv., P.T. 1861, are the stars 1 and 5 of my list; the objects *a* and *β* referred to in the observation of Nov. 20, 1862, correspond with the nebulous patches *b* and *c* on my diagram. As stated above, fig. 7 is oriented with *p* side up and *n* to the right.

The Nebulæ, N.G.C. 821, 2655, 2782, 2787, 2830, 2859, *and* 2880.

The above-mentioned nebulæ depicted on Plate 10 belong to the lists of nebulæ (for which descriptions are desired) communicated to me by Mr. A. R. Hinks, F.R.S.

The diagrams were made from preliminary measures with the Isaac Roberts instrument, and are represented in the illustrations accompanying this paper on the scale 3·5 mm. = 1′ of arc, or 1 mm. = about 17″. On the original I.R.* plates 1 mm. = 81″·5.

Fig. 3: N.G.C. 2880, G.C. 1848, H.I. 260, *h* 596, Ursæ Majoris.

I.R.* Reflector plate 1901. Date: March 26, 1895. Exp.: $8^h\ 59^m$ to $10^h\ 29^m$, S.T. Crowborough. ($1^h\ 30^m$).

R.A. $9^h\ 21^m\ 45^s$. Decl. 62° 56′·1 (1900).

Described in Sir J. F. Herschel's catalogue as B, *c*S, R, *mb*M, *umst.*

The photograph shows the nebula like a dense stellar nucleus (Int. 6), diameter about 13″, involved in extremely faint nebulosity (Int. 17), elongated *sf* to *np* in position angle 133° ± and measuring about 1′·5 (major axis), 0′·5 (minor axis). In the *sp* and *nf* regions of the nebula, at about 1′·5 from its nucleus, are very faint patches of nebulosity like faint arcs, to which the fainter nebulosity surrounding the nucleus seems to extend. The two very faint knots or extremely faint stars south of the nucleus, at X = 0′·0, Y = − 1′·2 (Int. 19) and at X = + 0′·6, Y = − 1′·4 (Int. 18) appear to be involved in the fainter exterior nebulosity; the extremely small star at X = + 1′·5; Y = − 0′·6 (Int. 16) appears to be at its *f* margin.

North of the nebula, at X = + 0′·4, Y = + 3′·3, is a very small dense stellar nucleus involved in faint nebulosity (Int. 14, 17) which appears like a small nebula.

A longer exposure of the nebula H.I. 260 is required to confirm the reality of the faint nebulous arcs mentioned above and shown on the diagram.

Fig. 4: N.G.C. 821, G.C. 487, H.I. 152, *h* 193, Piscium.

I.R.* Reflector plate 1761. Date: Dec 10, 1890. Exp.: from $1^h\ 12^m$ to $2^h\ 12^m$, S.T. Crowborough (60^m).

R.A. $2^h\ 3^m\ 1^s$. Decl. 10° 31′·1 (1900).

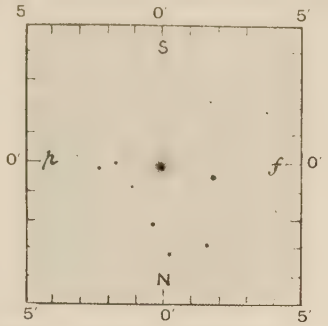


Fig. 3.—I.R. * 1901. H.I. 260 Ursæ Maj.

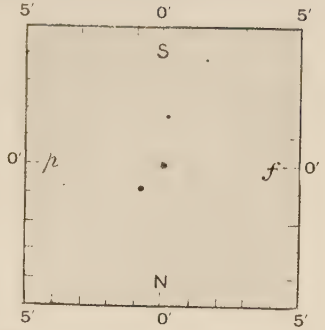


Fig. 4.—I.R. * 1761. H.I. 152 Piscium.

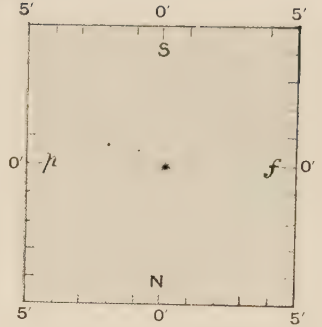


Fig. 5.—I.R. * 1877. H.I. 288 Draconis.

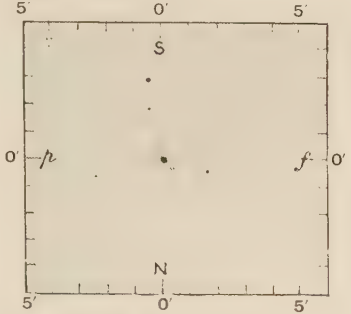


Fig. 6.—I.R. * 1891. H.I. 167 Lyncis.

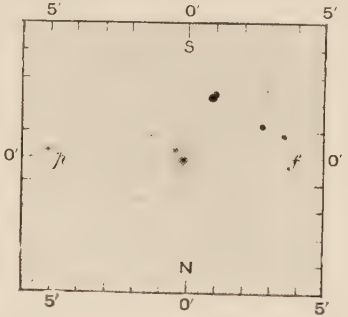


Fig. 7.—I.R. * 1895. H.I. 113 Lyncis.

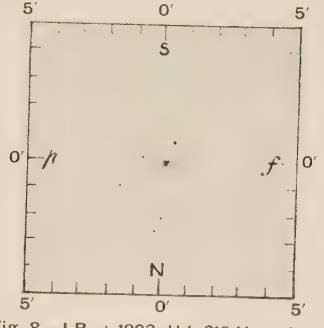


Fig. 8.—I.R. * 1893. H.I. 216 Ursæ Maj.

Described in Sir John Herschel's Catalogue as B, *v*S, *vl*E, *svmb*M; *10, 55″ 320°.

Observed at Birr Castle, Dec. 18, 1856, and Oct. 16, 1876. The nebula is described as *b*MN, E *spnf*, S* in *s* end. The position angles and distance in reference to the nebula are given for the two stars mags. 9 and 11, which on the diagram are respectively at

$$X = -0''.7,\ Y = +0''.8 \text{ (Int. 2)}$$
$$X = +0''.1,\ Y = -1''.7 \text{ (Int. 4)}.$$

The observations are accompanied by a sketch.

In Isaac Roberts' Register of Celestial Photographs, H.I. 152 is described as "a nebulous star involved in elongated nebulosity." The nucleus (Int. 7) is about 10″ in diameter, the surrounding nebulosity (Int. 14, 17), decreasing rapidly in intensity, presents near or beyond its margin the two small nebulous knots at $X = -0''.6$, $Y = -0''.8$ (Int. 18), referred to in the observation of Dec. 18, 1856, and $X = +1''.2$, $Y = -0''.4$ (Int. 19).

The very small star at $X = -1''.0$, $Y = 0''.0$ (Int. 9) and the star mag. 9, observed at Birr Castle, are near the *p* and *np* margin of the nebula. The nebula is elongated *sp* to *nf*.

A longer exposure is desired.

Fig. 5: N.G.C. 2655, G.C. 1691, H.I. 288, *h* 520, Draconis.

I.R.* Reflector plate 1877. Date: Jan. 13, 1890. Exp.: from $7^h\ 9^m$ to $9^h\ 10^m$, S.T. Crowborough. (2^h) [Interruptions one minute.]

R.A. $8^h\ 42^m\ 32^s$. Decl. 78° 35′·8 (1900).

Described in Sir John Herschel's Catalogue as *v*B, *c*L, *l*E 90° ±; *g*, *vsmb*M.

Observed at Birr Castle with the 3-ft. telescope on Feb. 10, 1861, and on March 21, 1862. In 1861, the observer defines the nebula "as very bright," and adds: "I think in the 6 ft. it would be resolvable."

The photograph shows the nebula with a stellar nucleus (Int. 3, diameters 11″ and 14″) involved in nebulosity (Int. 10 . . . 17), rapidly decreasing in intensity, elongated in position angle 88°, the denser part extending over 1′·6 major axis, and 0′·7 minor axis.

At $X = -1''.0$, $Y = -0''.6$ is a small star (Int. 5 . . . 10) to which the outlying nebulosity seems to extend. In the *np* and *sf* regions of the nebula appear to be extremely faint nebulous patches like very faint condensations in a curve.

The nebula appears to be a spiral, with extremely faint left-hand arms.

Another exposure is desired.

The small star in the *sp* region of the nebula is at $X = -2''.0$, $Y = -0''.6$ (Int. 3 . . . 10).

Fig. 6: N.G.C. 2782, G.C. 1778, H.I. 167, *h* 568, Lyncis.

I.R.* Reflector plate 1891. Date: March 25, 1895. Exp.: from $8^h\ 20^m$ to $9^h\ 50^m$, S.T. Crowborough.

R.A. $9^h\ 7^m\ 45^s$. Decl. 40° 31′·5 (1900).

Described in Sir John Herschel's Catalogue as *c*B, R, *mb*MBN.

Observed at Birr Castle on Feb. 26, 1851, and on March 9, 1874, on which date the nebula was described as B, *p*L, R, *lirr*, *pgsmb*MN. The position angles and the distances of the stars *a*, *b*, *c*, *d* are given, which on the diagram are respectively at:

*(*a*) X = +1′·5, Y = +0′·4 (Int. 7 . . . 15), $2a = 7''$, $2b = 6''$ mag. 12·5 (Lord Rosse).

The distance of *(*a*) from the nucleus is approximately 97′, whereas the distance given on page 73 of *Observations of Nebulæ and Clusters of Stars* is 35″·8.

*(*b*) X = −0′·4, Y = −1′·8 (Int. 6), $2a = 8''$, $2b = 5''$, mag. Lord Rosse 12·5.

(*c*) a triple star. Lord Rosse remarks that it looked like a nebula at first.

*c*1	X = −4′·0,	Y = −4′·2	(Int. 12 . . 17),	$2a = 5''$,	$2b = 5''$	
*c*2	−4·2,	−4·2	15 . . 17	6	4	
*c*3	−4·1,	−4·0	14 . . 17	5	5	
*d	−2·6,	+0·6	11 . . 14	8	6	mag. Lord Rosse, 12·5.

The I.R.* photograph shows the nebula H.I. 167 to be a very faint spiral with a dense stellar nucleus (Int. 4), $2a = 15''$, $2b = 12''$, slightly elongated (as the stars in the region of the nebula), and involved in faint nebulosity which is a little denser southward. On the *p* side of the nucleus at about X = 0′·5 is a short arc of faint nebulosity like diffused condensations or knots grouped along a curve; two fainter patches of nebulosity are north of N at

X = −0′·1, Y = +0′·4 (18)
+0′·3, +0′·4 (17),

both involved in very faint nebulosity.

The denser central region of H.I. 167 is lined by much fainter nebulosity extending, on the *p* side of N, over 1′, and on the *f* side over 2′·5, forming on the *f* side a very faint nebulous arc which encircles star (*a*).

In the *sf* region of H.I. 167 at X = +10′·8, Y = −4′·7 is a small faint nebula elongated in position angle 13° ±, $2a = 130'' \pm$, $2b = 24'' \pm$, with central condensation (Int. 16). The nebula is not in the N.G.C. On the annexed diagram it is placed at half of its distance from H.I. 167.

Fig. 7: N.G.C. 2830, G.C. 1811, H.I. 113, *h* 582, Lyncis.

I.R.* Reflector plate 1895. Date: Feb. 23, 1895. Exp.: from $7^h\ 12^m$ to $8^h\ 42^m$, S.T. Crowborough.

R.A. $9^h\ 13^m\ 41^s$. Decl. 34° 10′·3 (1900).

Described in N.G.C. as *c*B, *c*L, E, 1st of 3, the others being N.G.C. 2831, 2832.

The nebula H.I. 113, G.C. 1811, and the nebulæ G.C. 1804–5–6–7–8, G.C. 1810–1–2–3–4–5–7–8–1821 were observed at Birr Castle in the years 1850–51–55–58–61–67 and 1878. A sketch accompanies the observations.

On the I.R.* plate 1895 the group G.C. 1811–2–3–5–1816 is near the *sp* region of the plate, the nebulæ are diffused and the stars in their vicinity slightly distorted; another exposure with the group more centrally placed would be required.

The photograph shows H.I. 113 as small and faint (Int. 16 . . . 18), elongated in position angle 109°, and extending over 55″ and 17″ approximately. The nebula is in the *sp* region of the double nebula N.G.C. 2832–2831 (G.C. 1813–1812), of which it seems to form part.

The nucleus of the greater nebula is N.G.C. 2832, G.C. 1813, Lord Rosse *a*, described in N.G.C. as F, *v*S, R, 3rd of 3. It is about 11″ in diameter (Int. 9), surrounded by pretty dense nebulosity (Int. 16) extending over 34″. Small nebulous knots or very faint stars are at

X = −0′·1, Y = +0′·6 (Int. 18)
+0′·5, −0′·4 (19)

perhaps involved in extremely faint nebulosity.

The small nebula *β* Lord Rosse, G.C. 1812, N.G.C. 2831, described as F, S, *l*E, *b*M, 2nd of 3, is at X = −0′·3, Y = −0′·3, Int. 14, like a small secondary nucleus or nebulous condensation, $2a = 14''$, $2b = 12''$, subordinate to N.G.C. 2832, and involved in faint nebulosity.

At greater distances from the central nucleus are the following:—

At X = −1′·4, Y = +1′·6 (Int. 19), a nebulous knot or very faint star,
−1′·2 −0′·7 H.I. 113,
−0′·3 −1′·8 (Int. 19), a nebulous knot or very faint star.

At still greater distances are the small nebulæ N.G.C. 2825 and 2834:—

X = −5′·0, Y = −0′·2, N.G.C. 2825, G.C. 1806, *h* 581, described in N.G.C. as F, *p*S, *l*E, *b*M, which appears on the plate as a small nebulous nucleus (Int. 15 . . . 17 . . . 19), diameters 22″ 19″. Lord Rosse *δ*.

X = +3′·1, Y = −2′·5, N.G.C. 2834, G.C. 1815, Lord Rosse *ϵ*, described in N.G.C. as *v*F, S, R, *b*M, which appears on the I.R.* plate 1895 like a small faint nebulous nucleus (Int. 17, 18), elongated like the stars in its vicinity.

The dense stars marked on the annexed diagram are approximately at

X = +1′·0, Y = −2′·4 (Int. 2, 14) $2a = 26''$ $2b = 24''$

double star with images overlapping

X = +2′·9, Y = −1′·2 (Int. 4) $2a = 21$ $2b = 18$
X = +3′·4, Y = −0′·9 (Int. 5, 10) $2a = 19$ $2b = 15$

Another exposure of the field is desired, in particular of the region considered above.

Fig. 8: N.G.C. 2787, G.C. 1781, H.I. 216, *h* 570, Ursæ Majoris.

I.R.* Reflector plate 1893. Date: March 24, 1895. Exp. from $8^h\ 53^m$ to $10^h\ 10^m$, S.T. Crowborough.

R.A. $9^h\ 10^m\ 15^s$. Decl. 69° 37′·2 (1900).

Described in N.G.C. as B, *p*L, *l*E 90°, *mb*M, *r*, vS*, *sf*, *inv.*

The photograph shows the nebula H.I. 216 with a small dense stellar nucleus (Int. 4), diameter about 7″, involved in pretty dense nebulosity, elongated in position angle 111°, which forms the central region of the nebula and covers about 31″ and 16″ (Int. 7 . . . 14). Much fainter nebulosity (Int. 17 . . .), elongated, extending over more than 3′ *np* to *sf* (major axis) and over 1′ (minor axis), surrounds the denser region and seems to be bordered toward the *s* and *p* sides by the three small stars at

X = +0′·3,	Y = −0′·8 (Int. 4 . . . 14),	diam.	8″·7″
−0′·9	−0′·2	15	7 ·6
−1′·7	+0′·8	13	7 ·5 .

To the north of the nebula is a star at

X = −0′·0, Y = +2′·0 (Int. 12), diam. 7″·5″,

with two very faint knots or very faint stars *np* at

X = −0′·4, Y = +2′·5 (Int. 16).
−0′·4 +2′·2 (Int. 18).

nf the nebula (not marked on the diagram) at about X = +10′·4, Y = +11′·6, is a very faint nebula, elongated *f* to *p*, covering about 100″ (major axis), 11″ (minor axis), and presenting a faint nebulous nucleus. An area almost void of stars separates this small nebula from H.I. 216.

Another exposure is desired.

The I.R.* plate 1895 mentioned above contains the photographic image of the planetary nebula

N.G.C. 2859, G.C. 1837, H.I. 137, *h* 593, Lyncis,

which presents certain points of resemblance with H.IV. 69 Tauri and is depicted on Plate 8, fig. 2.

H.I. 137 has a dense stellar nucleus (Int. 7), diameters 23″ 20″, surrounded by nebulosity (Int. 17, 15), which increases in intensity southward and northward and measures about 1′·7 *n* to *s* and 2′ *f* to *p*. Near its *s* margin is a nebulous arc (Int. 16), which seems to be formed by very faint nebulous condensations, the densest being at

X = +0′·2, Y = −0′·4 (Int. 16).

North of the nucleus, involved in very faint nebulosity, are two diffused nebulous knots or condensations at

$$X = -0'\cdot1,\ Y = +0'\cdot4 \text{ (Int. 16)},$$
$$X = +0'\cdot2,\ Y = +0'\cdot7 \text{ (Int. 16)},$$

which give the nebula the appearance of a broken annulus or a faint right-hand spiral with two arms.

In the *np* region of the nebula, at $X = -0'\cdot4$, $Y = +0'\cdot9$, is a faint star or nebulous knot (Int. 16) situated beyond the denser nebulosity which surrounds the nucleus of H.I. 137.

In its *f* and *p* regions the nebula is extremely faint and not so well defined as *n* and *s*.

The small star *sf* the nebula at $X = +1'\cdot0$, $Y = -1''\cdot4$ (Int. 14) appears free from nebulosity.

A denser star is at $X = +2''\cdot6$, $Y = +1'\cdot0$ (Int. 5 . . . 14).

nf, the nebula at $X = +21''\cdot2$, $Y = +8''\cdot2$, is a very faint nebula elongated *nf* to *sp* (Int. 17, 18), and measuring about $2a = 1''\cdot7 \pm 2b = 0'\cdot4$, which is mentioned by the Birr Castle observer of March 28, 1861.

A fainter nebula, elongated *s* to *n* (Int. 18), and measuring about $2a = 0'\cdot7$, $2b = 0'\cdot3$, is at $X = +20'\cdot6$, $Y = +6'\cdot7$.

The small star north of the faint nebula observed at Birr Castle is at

$$X = +20'\cdot3,\ Y = +9'\cdot1 \text{ (Int. 5 . . . 15) } 2a = 14'',\ 2b = 12''.$$

On the diagram these three objects are placed at half their distance from H.I. 137.

H.I. 137 was observed at Birr Castle in 1855–7–61 and 1876, and is described as follows:—

1855. Feb. 14. "Like *h* 476, *sb*M, stellar points in outlying faint nebulosity, especially two, which I can plainly see at moments with $\frac{1}{2}$-inch single lens."
1857. Feb. 23. "Looks very like a cluster, but I cannot resolve it. 2F *st* on its edge."
1861. Jan. 15. "R, *v*B nucl., fades off rapidly."
1861. March 28. "Dull nucl., I suspect a dark ring round it. A *v*F neb. *nf*. I strongly suspect stars in *h* 593."
1876. March 17. "*p*B, *p*S, *vsmb*M. . . ."

h 476, H.III. 750, G.C. 1604, N.G.C. 2493, is not in the Isaac Roberts Collection.

Maison de Rosa Bonheur,
By-Thomery:
1913 *December* 8.

Micrometrical Measures of Double Stars (Tenth Series) and New Double Stars. By the Rev. T. E. Espin, M.A.

Name.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	Date.
	h m	° ′	°	″			1913.
ρ 59	0 8·6	46 15	212·5	11·73	9·5 10·2	3	·951
ρ 60	25·0	46 4	299·7	11·68	9·3 10·6	3	·951
ρ 61	27·2	46 10	98·8	12·50	9·1 9·6	3	·951
+49° 131	29·2	49 40	67·6	9·58	7·7 14·0	3	·860 1912
+46° 144	37·4	46 48	67·6	12·27	9·1 9·2	2	·935
h 2033	1 11·9	48 30	51·7	10·17	9·5 11·3	2	·956
Σ 112	14·8	45 50	187·0	4·75	8·3 12·0	3	·984 BC
			331·7	22·18	8·0 8·3	4	·970 AB
+47° 451	28·2	48 14	14·2	7·62	9·4 12·0	2	·074
h 2057	30·1	45 51	44·5	11·37	8·7 10·1	2	·969
A.G. 24	45·5	45 19	171·8	16·05	8·5 8·7	2	·967
+48·865	3 7·3	48 12	195·8	11·65	8·5 9·2	2	·034
β 1294	13·8	46 19	229·8	6·17	8·8 9·1	2	·962
Σ 384	20·4	59 33	266·8	1·95		2	·109
Holmes	21·6	59 34	51·2	5·12	9·0 10·5	2	·109
Webb	34·5	59 39	94·9	21·37	7·5 13·0	3	·129 AB
			300·0	34·90	C= 12·6	3	·129 AC
			36·0	54·28	D= 8·5	3	·129 AD
Σ 519 rej.	4 13·3	50 8	346·7	17·97	7·2 9·3	3	·173
OΣΣ 47	13·4	50 1	257·8	22·80	7·5 10·5	3	·173 BC
			326·3	26·08	*a*= 14·0	3	·173 A*a*
			327·6	72·55	6·5 7·5	3	·173 AB
Σ 586 rej.	39·1	52 57	129·3	28·40	9·4 9·8	1	·189
Σ 587	40·1	52 56	184·4	21·15	6·5 8·8	1	·189
Σ 619	53·6	50 7	125·0	4·03		3	·175
Webb	6 2·5	43 9	216·1	43·63	6·9 8·4	3	·172
+43° 1475	4·3	43 11	214·8	15·42	6·8 10·5	2	·169
Σ 1009	57·7	52 54	154·3	3·35	6·7 6·8	2	·187
+46° 1255	7 14·2	46 25	265·5	81·10	8·0 8·2	1	·257
+46° 1476	37·2	45 36	277·7	11·10	8·0 11·5	1	·148
Espin 591	8 0·5	45 30	48·2	2·02	9·5 9·8	2	·235
A 1774	3·9	44 57	300·1	4·13	9·0 10·8	3	·220
+45° 1727	9 24·2	45 28	239·9	36·40	9·0 9·3	2	·235
Σ 1358	24·5	45 7	163·3	24·32	7·3 8·5	2	·235
Σ 1376	38·7	43 41	132·7	5·20	8·2 8·4	2	·235 AB
			331·9	68·88	C= 13·5	2	·235 AC
+44° 1913	44·0	44 28	275·2	12·75	8·0 9·5	1	·307 1912
+45° 1848	10 32·2	45 24	271·4	7·52	9·2 11·5	2	·267 1912

Name.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	Date.
	h m	° ′	°	″			1913.
+44° 2024	10 47·4	44 31	45·4	8·70	8·5 9·3	2	·305 1912
+43° 2069	55·0	43 16	200·1	13·50	7·0 11·0	2	·297
OΣ 240	11 48·4	43 28	318·9	8·40	8·0 9·6	2	·349 1912
A 1778	52·6	45 51	226·9	2·77	8·7 10·4	3	·287 1912
+44° 2162	12 3·3	44 28	278·8	19·20	8·1 9·3	2	·371 1912
Espin 124	11·2	42 27	116·6	7·98	8·5 12·5	4	·387
h 1215	11·6	42 23	18·5	26·37	10·0 10·1	3	·384
h 2614	26·9	41 7	33·0	23·57	10·0 11·2	2	·387
A 1850	36·5	44 32	50·2	2·60	8·9 10·1	2	·371 1912
+43° 2261	44·5	42 47	50·1	47·48	7·3 7·6	3	·374
Espin 606	56·8	42 58	283·2	9·25	8·3 10·5	1	·375
Espin 607	13 0·7	43 15	224·7	6·10	9·0 10·5	2	·398
Espin 125	0·9	42 13	121·9	2·67	8·0 10·2	2	·395
h 1230	11·9	42 34	168·1	16·30	10·5 11·0	2	·387
+43° 2335	22·1	43 4	219·1	24·35	9·5 9·6	2	·387
h 1247	14 6·6	41 35	133·2	5·52	9·0 9·8	3	·426
θ Boötis (Ward)	21·8	52 19	181·9	68·80	4·0 11·5	2	·431
Ku 49	40·6	41 50	199·8	1·55	9·2 9·7	1	·443
h 1258	46·9	43 51	69·4	25·25	9·1 13·0	2	·430
Σ 1895	53·7	40 34	42·9	12·57	7·2 7·7	2	·424
h 1264	54·2	40 37	318·4	19·40	9·2 9·4	3	·433
h 2773	15 13·3	41 47	145·1	28·42	8·9 9·5	2	·438
Espin 74	13·4	41 39	121·7	8·87	8·0 12·0	2	·438
+45° 2280	18·8	45 29	5·6	26·22	8·6 8·9	2	·424
+45° 2470	16 53·5	45 14	251·9	48·70	8·2 8·3	2	·505
A 1881	17 41·3	46 16	204·8	4·46	9·1 10·7	3	·598
+44° 2878	18 16·8	44 51	21·9	10·23	8·2 10·1	3	·754
+45° 2742	31·4	45 6	227·1	11·65	8·8 10·6	3	·658
Σ 2380	40·0	44 49	8·7	25·98	7·2 8·4	3	·717
+44° 2985	41·8	44 51	50·1	10·80	8·6 9·0	3	·754
Espin 22	41·8	+45 44	134·2	2·85	9·2 12·5	3	·804 AB
			216·0	24·55	9·8	2	·796 AC
h 1356	53·5	45 23	342·3	29·88	8·6 8·9	2	·804
h 1357	54·2	45 44	212·2	27·57	8·0 9·7	2	·810
A.G. (Bonn)	55·3	47 3	229·7	2·35	8·3 8·4	4	·775
Σ 2463	19 3·1	45 40	5·6	10·07	8·4 9·1	3	·652 AB
			281·7	24·30	C= 12·3	3	·652 AC
h 1380	9·9	47 35	227·7	4·95	8·9 9·3	3	·797
Espin 128	14·0	47 0	279·7	4·30	8·5 12·0	2	·796
Espin 23	47·2	44 54	136·5	8·47	7·5 11·6	5	·724
			327·0	31·50	C= 9·0	3	·704
OΣ 393	54·6	44 7	228·6	20·12	7·5 8·5	2	·690
o^1 Cygni (Espin 26)	20 10·2	46 31	250·7	34·32	5·0 13·2	3	·874 1912

Name.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	Date.
	h m	° ′	°	″			1913.
Σ 2658	20 11·0	52 49	114·0	5·15	7·0 9·0	2	·697 AB
			311·9	33·87	*a* = 13·0	2	·697 A*a* (New)
			209·3	46·00	C = 8·8	2	·697 AC
Espin 27	11·2	46 34	339 2	4·10	9·3 9·3	2	·909 1912
Σ 2707	34·6	47 35	30·7	22·42	7·2 8·7	2	·936 AB 1912
			195·9	54·57	C = 7·5	2	·936 AC
			191·2	16·35	*a* = 13·5	2	·936 A*a* (New)
Espin 89	35·2	47 43	199·5	16·60	6·5 10·2	2	·936 1912
+44° 3596	46·5	44 58	346·0	9·85	8·2 13·2	3	·800
Arg. 40	47·9	44 56	251·6	8·62	8·8 9·1	2	·796
59 Cygni	56·4	47 7	353·8	19·93	4·5 8·0	3	·282 AB
			141·2	26·58	9·0	3	·282 AC
			220·3	38·27	11·0	3	·282 AD
β 160	21 8·5	45 19	111·0	6·40	10·7 10·9	2	·735 BC
			154·3	57·47	A = 7·4	3	·735 AB
h 1635	15·0	47 22	360·0	15·48	9·2 11·2	3	·923 AB 1912
			82·8	18·87	C = 13·4	2	·931 AC
			240·0	18·62	D = 13·8	2	·931 AD
h 1660	30·7	45 32	239·7	5·97	11·6 12·2	2	·924
Espin 101	32·7	45 42	3·2	3·35	9·5 11·7	3	·939
Espin 34	33·6	50 5	136·4	2·77	9·0 11·7	2	·909 AB 1912
			69·8	39·50	C = 9·5	2	·909 AC
Σ 2832	45·6	50 3	213·4	12·72	7·0 7·3	3	·940 1912 AB
			319·1	45·92	C = 9·7	3	·940 1912 AC
			333·5	51·13		3	·940 1912 BC
A 780	57·3	44 46	146·9	1·67	7·5 7·7	2	·941 AB
h 1715	57·9	44 48	252·7	13·65	11·0 11·7	3	·939
β 700	22 22·4	49 11	335·1	9·80	8·2 12·0	2	·896 1912
β 380	22·8	49 12	324·0	24·87	7·4 11·7	2	·896 1912 AB
			135·1	36·70	C = 7·6	2	·896 1912 AC
			242·6	21·27	D = 12·5	2	·896 1912 CD
			89·5	23·60	*a* = 13·3	2	·896 1912 A*a*
			26·9	17·77	*c* = 13·6	2	·896 1912 C*c*
+49° 4088	23 15·2	50 8	292·7	7·20	9·5 12·5	4	·806 1912
+50° 4101	34·1	50 21	267·4	7·40	9·5 13·0	2	·762 1912
A.G. (Bonn)	35·3	49 15	245·1	18·25	8·7 8·8	2	·799 1912
Hu 794	35·6	49 22	67·8	2·60	8·9 11·4	2	·799 1912
+49° 4189	36·4	49 20	244·2	13·05	9·1 9·5	2	·799 1912
h 1903	37·5	49 28	207·1	6·27	9·5 9·6	2	·799 BC 1912
			9·4	28·27	A = 9·3	2	·799 AB 1912
			195·2	11·4	*a* = 13·5	1	·805 A*a* 1912
ρ 58	46·6	46 30	65·5	5·27	9·5 9·7	2	·386

Notes.

OΣΣ 47.—The only other measures of BC were made at the time of its being first seen—1892·96, P 257°·0, D 21″·68, 2 nts. The small *comes a* has not been observed before.

Webb, 6h 2m·5.—Taken from Mr. Webb's MS. His observation, 220°±, 35″±, 7·5, 9·5; 1878 April 6. The chief star is B.D. +43°, 1466. I have measured also B.D. +43°, 1475, a similar pair in angle, but with *comes* fainter, and distance a third less.

A. G., 18h 55m·3.—This fine pair is not in Burnham, and the only other measures are mine—1907·85, P 229°·0, D 2″·37, 3 nts.

o^1 Cygni (B.D. +46°, 2881).—The only other measures are mine—1900·48, P 252°·0, D 32″·92, 4 nts.

β 160.—Two faint *comites* follow BC, nearest P 84°·6, D 23″·2, mag. 13, from single setting.

h 1635, 21h 15m, *h* 13°·0.—As the angle differed from *h*, the place was redetermined, and leaves no doubt that the right star was observed.

h 1660, 21h 30m·7, *h* 217°·8.—The place of this star was redetermined and found to agree with that of *h*. There are no other measures.

h 1715, 21h 57m·9, *h* 251°·8.—There are no other measures.

β 380.—The small *comites a* and *c* are mentioned by β.

A.G. (Bonn), B.D. +49°, 4183.—This star was measured by β for *h* 1903.

h 1903, 23h37′·5.—This star is B.D. +49°, 4195. *h*'s angle 295°·5, and he adds "a star, 10 mag. rather to the south of the line of junction." The very faint *comes a* is new, and nothing more was obtained except a single setting.

ρ 58.—The neighbouring star was found to be a somewhat similar pair in distance, and is No. 1291 in the list of new doubles.

New Double Stars.

No.	B.D.	R.A. 1900. h	m	Decl. °	′	P. °	D. ″	Mags.		Nts.	Date. 1913.	
1192	+44° 4551	0	0·3	+44	48	298·9	1·70	9·2	9·4	3	·992	
1193	+46° 4261		2·0	46	50	70·7	2·60	9·5	9·6	2	·796	
1194	+45° 28		8·0	46	3	290·1	5·77	9·5	10·2	2	·976	
1195	+45° 36		9·0	45	39	11·0	7·25	9·3	9·6	3	·999	
1196	+45° 59		14·4	46	6	184·5	5·65	9·5	13·2	2	·990	
1197	+48° 83		15·2	48	38	107·2	6·30	8·5	11·5	3	·904	
1198	+48° 88		15·9	48	15	188·1	6·78	8·6	11·5	3	·931	
1199	+45° 74		17·4	45	41	16·1	2·97	9·2	9·6	3	·999	
1200	+45° 76		18·0	46	0	55·2	1·75	13·0	13·3	3	·991	BC
						325·1	47·75	A=	8·2	2	·987	AB
1201	+45° 80		18·4	46	1	75·6	7·05	8·5	12·8	3	·999	
1202	+45° 108		22·9	46	12	33·4	2·52	9·6	10·0	2	·987	
1203	+46° 138		36·2	46	54	66·6	6·87	8·3	11·5	2	·935	
1204	+48° 304		53·3	48	41	348·2	4·09	9·6	9·8	3	·840	
1205	+48° 316		55·5	48	20	113·6	2·87	9·3	9·8	2	·825	
1206	+47° 304		59·7	47	56	179·6	5·45	8·8	12·7	3	·974	
1207	+48° 386	1	10·6	48	33	95·8	4·00	9·1	12·0	2	·957	
1208	+48° 389		11·6	48	27	51·1	5·47	9·5	10·5	2	·957	
1209	+48° 420		18·5	48	42	308·2	5·92	9·6	9·8	2	·879	
1210	+46° 367		23·9	46	59	180·7	2·35	9·2	11·2	2	·981	

No.	B.D.	R.A. 1900.	Decl.	P.	D	Mags.	Nts.	Date.	
		h m	° ′	°	″			1913.	
1211	+46° 426	1 35·8	46 15	235·5	6·66	8·5 13·0	3	·991	
1212	+48° 529	41·7	48 19	13·1	4·06	9·5 9·6	3	·056	
1213	+46° 470	46·9	46 51	131·2	2·41	9·4 12·0	3	·955	
1214	+46° 481	50·8	46 24	215·1	3·35	9·4 9·6	2	·965	
1215	+47° 576	2 4·7	47 31	172·8	3·45	9·3 12·4	3	·846	
1216	+46° 583	25·9	46 32	92·8	1·62	9·3 9·4	3	·958	
1217		48·9	47 22	282·5	2·33	9·4 9·5	3	·966	
1218	+45° 759	3 18·5	45 24	263·0	1·72	9·6 9·9	3	·991	
1219	+59° 663	22·8	59 33	271·1	5·52	9·4 12·7	2	·160	
1220	+59° 664	22·8	59 34	346 1	3·77	9·3 13·2	2	·160	
1221	+49° 1009	39·6	49 23	1·9	2·57	9·4 9·7	3	·055	
1222	+48° 1014	46·1	48 23	87·8	5·82	9·4 10·1	2	·976	
1223	+47° 941	4 1·8	47 45	111·5	4·05	9·4 11·3	3	·979	
1224	+47° 951	4·7	48 1	188·7	1·45	9·5 9·6	2	·976	
1225	+47° 987	16·4	47 27	130·8	5·15	9·5 10·7	2	·991	
1226	+48° 1126	33·5	48 28	276·8	7·32	9·0 10·7	3	·185	
1227	+47° 1063	46·3	47 26	237·4	4·10	9·3 10·7	3	·987	
1228	+48° 1186	50·4	48 53	302·1	6·52	9·2 9·8	2	·108	
1229	+48° 1211	57·4	48 23	256·2	5·66	9·3 10·7	4	·169	
1230	+47° 1098	5 0·3	47 40	6·2	7·30	9·0 12·2	2	·183	
1231	+46° 1002	15·1	46 15	35·9	1·12	9·3 10·0	2	·994	AB
				9·4	14·12	C= 10·5	2	·994	AC
1232	+47° 1224	52·2	47 51	262·7	3·75	9·5 10·7	2	·183	AB
				175·2	22·72	C= 9·8	2	·183	AC
1233		56·3	47 54	211·1	3·03	9·2 11·0	3	·185	
1234	+48° 1339	58·3	48 15	267·0	9·82	7·1 11·5	2	·169	
1235	+47° 1289	6 14·0	47 24	156·2	2·72	9·4 9·5	4	·211	
1236	+47° 1291	16·0	47 21	266·1	6·02	8·9 10·8	2	·208	
1237	+48° 1433	39·8	48 32	358·0	5·25	9·0 11·2	3	·177	
1238	+47° 1376	52·8	47 17	202·1	3·25	9·6 9·9	2	·235	
1239	+46° 1212	53·1	46 35	0·0	5·10	9·4 9·7	1	·257	
1240	+47° 1402	7 0·6	47 0	102·6	7·92	9·3 12·8	3	·352	AB
				285·0	11·25	C= 13·9	3	·352	AC
1241	+48° 1514	12·8	48 0	0·5	3·57	9·4 9·4	2	·177	
1242	+46° 1265	19·1	46 23	268·7	7·85	9·2 9·5	1	·257	
1243	+46° 1295	33·4	46 5	320·2	3·25	9·4 10·1	2	·250	
1244	+43° 1965	9 44·1	43 13	227·2	2·05	9·1 10·1	2	·274	
1245	+43° 1986	55·7	43 17	169·9	5·55	8·8 9·7	3	·343	
1246	+43° 2039	10 36·7	43 35	56·5	3·67	9·3 9·7	2	·297	
1247	+43° 2046	42·3	43 27	281·8	2·72	9·1 9·6	2	·250	

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	Date.
		h m	° ′	°	″			1913.
1248	+42° 2277	12 6·2	41 54	76·9	2·07	9·5 9·7	2	·373
1249	+43° 2313	13 9·3	43 38	201·6	2·87	9·5 12·0	3	·411
1250	+42° 2519	14 35·8	42 18	164·7	6·45	8·8 12·2	2	·431
1251	+40° 2832	54·0	40 35	54·5	7·03	8·7 13·1	3	·431
1252	+46° 2051	15 14·4	46 33	20·2	1·95	9·5 9·6	3	·397
1253	+45° 2392	16 15·4	45 24	231·6	6·40	9·2 10·2	2	·424
1254	+45° 2476	55·0	45 32	133·4	5·30	9·5 12·5	3	·481
1255	+46° 2253	58·1	46 25	44·8	8·41	8·0 11·7	4	·496
1256	+47° 2486	17 24·7	47 21	332·3	5·03	9·4 12·4	3	·576
1257	+45° 2574	35·4	45 3	261·1	2·70	9·1 9·5	3	·561 AB
				121·5	52·83	C= 9·6	2	·586 A
1258	+45° 2575	36·0	45 26	323·4	5·92	9·2 13·1	4	·598
1259	+46° 2581	49·4	46 58	226·7	6·25	8·6 9·3	4	·578
1260	+45° 2640	57·5	45 48	201·1	3·65	9·5 10·6	2	·606
1261	+45° 2735	18 29·5	45 6	207·8	8·07	8·9 11·5	2	·641
1262	+45° 2737	29·6	45 8	257·6	1·55	9·4 9·5	2	·641
1263	+46° 2538	41·0	46 13	242·3	6·20	9·5 12·0	3	·598
1264	+45° 2781	47·9	45 35	116·3	3·70	9·5 10·6	2	·619
1265	+45° 2793	51·5	46 2	299·2	5·55	9·5 12·6	3	·585
1266	+46° 2743	19 35·1	46 21	344·7	3·20	9·6 10·0	3	·580
1267	+46° 2795	49·3	46 31	161·7	5·78	8·8 12·3	3	·642
1268	+44° 3379	20 9·0	44 47	154·2	7·85	8·8 10·0	2	·796
1269	+44° 3380	9·1	44 51	46·5	5·72	9·0 13·2	2	·796
1270	+44° 3549	39·3	44 59	291·2	6·10	9·3 9·9	4	·626
1271	+44° 3672	57·5	45 4	30·7	2·95	9·4 10·1	2	·686
1272	+45° 3443	21 8·4	45 25	53·7	3·10	9·3 9·6	3	·765
1273	+45° 3446	8·9	45 27	73·2	4·77	13·1 13·6	2	·730 BC
				9·0	38·20	A= 8·6	4	·766 AB
1274	+45° 3556	25·5	46 2	240·2	2·55	9·4 13·3	3	·717
1275	+45° 3640	38·5	45 49	336·5	4·30	13·0 13·4	3	·813 BC
				279·2	5·22	D= 13·2	2	·805 BD
				3·6	66·47	A= 8·6	2	·805 AB
1276	+45° 3641	39·2	45 22	287·2	2·40	9·4 10·2	2	·820
1277	+47° 3748	22 11·8	48 9	5·9	6·05	9·2 12·8	3	·326 AB 1912
				199·5	11·72	C= 13·8	3	·326 AC 1912
1278	+45° 3891	16·5	45 21	73·8	8·12	8·5 10·7	2	·950
1279	+46° 3671	17·5	46 34	299·4	4·87	8·4 12·1	3	·904
1280	+45° 3907	18·5	45 16	255·4	6·42	8·6 9·2	2	·923
1281	+46° 3688	20·5	46 50	167·2	6·67	8·5 9·7	3	·856
1282	+45° 3918	20·6	45 19	55·8	4·51	9·0 12·3	3	·941

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	Date.
		h m	° ′	°	″			1913.
1283	+46° 3689	22 20·7	47 7	13·7	7·57	9·1 13·2	2	·805
1284	+46° 3697	21·7	46 53	65·7	2·09	9·3 12·0	4	·864
1285	+46° 3704	22·8	46 38	135·7	4·40	9·4 10·7	2	·818
1286	+45° 3982	29·2	45 21	352·6	3·07	9·2 10·1	2	·918
1287	+45° 4023	37·9	46 0	24·0	6·29	9·6 10·3	3	·905
1288	+45° 4024	38·0	46 4	338·2	4·90	9·4 10·4	3	·905
1289	+45° 4089	51·1	46 7	98·8	6·72	8·6 13·3	3	·852
1290	+45° 4166	23 8·0	46 13	65·7	3·15	9·4 10·5	2	·823 AB
				292·8	17·40	C = 11·6	2	·823 AC
1291	+46° 4194	46·9	46 29	146·8	5·17	9·1 10·7	2	·386
1292		56·6	45 44	86·7	3·10	9·7 9·8	2	·823
1293	+46° 4249	59·4	46 50	4·6	6·05	9·4 9·6	3	·800

Notes.

1200 BC.—A difficult object, and the measures are little better than approximate.

1208.—This pair lies SP Σ 102, while *h* 2033 lies NF; the angle of position is similar, but the distance is only about half that of *h* 2033.

1251.—Star in field immediately f Σ 1895, a very difficult object to measure in the twilight of the nights in June.

1273.—There is a 14 mag. P 290° ± D 24″ from A.

1279.—8·4 is orange-red.

On the Enhanced Lines of Manganese in the Spectrum of α Andromedæ. By F. E. Baxandall, A.R.C.Sc. (Plate 11.)

(*Communicated by Professor Newall.*)

In the Harvard publications relating to the classification of stellar spectra α Andromedæ is placed in Group VIII. (type star α Geminorum) in Miss Maury's classification, and in Class A_0 (type star α Canis Majoris) in Miss Cannon's later classification. In each case a suffix is given indicating that the spectrum shows peculiarities which distinguish it from the normal stars of the group. No detailed record of the lines in the spectrum appears to have been given either in the Harvard publications or elsewhere. In the former, however, the following general remarks are made * on the spectrum: "This star is peculiar in the intensities and wave-lengths of some of its lines. It may for convenience be classed with stars of Group VIII., approaching Group VII., but in the details of its spectrum it is not like those of either group. The hydrogen lines are the same as in the above-mentioned groups, and the majority of the remaining lines are those of Group VIII.,

* *Annals Harv. Coll. Obs.*, vol. xxviii., part i. p. 95. Remark 67.

Division *a*. Many of the solar lines which are present in that group, however, are absent, especially in the region having wave-length shorter than 4101·8, where the lines are few and weak. On the other hand, some well-marked lines appear which are faint or absent in normal stars of the group."

In a paper "On Some Stars with Peculiar Spectra," * by Sir Norman Lockyer and F. E. Baxandall, the following statement occurs with respect to the spectrum of α Andromedæ: "Next to the lines of hydrogen and helium the most prominent lines which have been traced to known terrestrial elements are those of silicium, Group II. (λλ 4128·20, 4131·04), proto-magnesium (λ 4481·3), and proto-calcium (λ 3933·83). The strongest enhanced lines of iron, chromium, carbon, and titanium are present, but occur only as comparatively weak lines. In addition to these lines of known origin, there occur several strange and well-marked lines, not found in the spectra of any other star yet examined, and for which no satisfactory terrestrial origin has yet been found. Of the strange lines those at λλ 3943·9, 3984·1, 4137·0, 4206·3, and 4282·4 are the most prominent."

In a later paper by R. H. Baker, "On the Orbit of α Andromedæ," † a list of eleven lines is given, on the measures of which lines the discussion of the orbit is based. Amongst these are four unidentified lines at λλ 3943·919, 3984·109, 4137·061, and 4206·211, which he refers to in the text as being "presumably blends." It will be seen that these are identical with four of the five lines previously mentioned.

It has now been found that these lines (except λ 3984·1), and numerous others which make the spectrum of α Andromedæ peculiar, agree so closely in wave-length with enhanced lines of manganese that there is little doubt that the two sets of lines are identical, and that the increased prominence of the proto-manganese lines, as compared with their occurrence in other stellar spectra, accounts to a great extent for the strangeness of the α Andromedæ spectrum. A comparison of the two sets of lines is given in the table on p. 252.

The first three columns give the wave-lengths and spark and arc intensities of the enhanced manganese lines. These are, in the main, as recorded in "Tables of Wave-Lengths of Enhanced Lines," published by the Solar Physics Committee in 1906. Those marked * were not recorded in that list, but have since been found in the best Kensington photograph (taken with a 6-inch concave Rowland grating) of the manganese spark spectrum. Those marked † are beyond the limits of the grating photographs, but occur in a manganese spark photograph obtained with a four-prism Steinheil spectrograph. The fourth, fifth, and sixth columns give the estimated wave-lengths of the α Andromedæ lines from measurements made by W. E. Rolston, F. J. M. Stratton, and F. E. Baxandall respectively. Of these, the first are the means of

* *Roy. Soc. Proc.*, vol. lxxvii. p. 551.
† *Allegheny Obs. Pub.*, vol. i., No. 3, p. 17.

Comparison of Enhanced Manganese Lines with Lines in α Andromedæ.

Manganese Enhanced Lines.			α Andromedæ Lines.						Diff. in λ *p* Mn − Star.	Remarks.
	Intensity.		Wave-Lengths.				Intensity.	Description of Line.		
λ.	Spark.	Arc.	W.E.R.	F.J.M.S.	F.E.B.	Weighted Mean.				
*3943·93	2	0	3943·83	...	3943·90	3943·86	4	Well defined	+0·07	...
4000·20	2	0	4000·15	...	...	4000·15	1–2	Weak and hazy	+0·05	...
4105·05	3	0	4105·49	...	...	4105·49	? 2	Hazy	−0·44	Difficult. Inside wing of Hδ.
4128·36	2–3	trace	4128·30	...	4128·20	4128·26	4–5	Well defined	+0·10	Stellar line chiefly due to Si 4128·20.
4137·16	3–4	0	4137·20	4137·82	4136·95	4137·05	3–4	Well defined	+0·11	...
4200·40	2	0	4200·62	4200·99	4201·06	4200·85	1–2	Hazy and broad	−0·45	Diffuse and indefinite line, measured as double line in Cambridge four-prism plate.
4206·56	4	0	4206·49	4206·68	4206·73	4206·61	3–4	Well defined	−0·05	...
4242·45	4	0	4242·54	4242·34	4242·59	4242·50	2	Well defined	−0·05	Stellar line partly due to *p* Cr 4242·54.
4244·43	1–2	0	4244·76	...	...	4244·76	1	Weak and indefinite	−0·33	...
4248·10	1	0	4247·72	4248·33	4247·72	4247·87	2	Rather hazy	+0·23	...
4251·86	5	0	...	...	...	...	...	...	...	Probably merged into neighbouring stellar lines, as it falls on an apparently continuous but narrow band in the stellar spectrum.
4253·13	5	0	4253·23	...	4253·62	4253·43	2	Rather hazy	−0·30	Difficult, merges into other spectrum lines.
4259·35	4	0	4258·73	4259·31	4259·32	4259·02	2–3	Well defined	+0·33	...
*4282·65	3	1–2	4283·07	4282·28	...	4282·87	2–3	Well defined	−0·22	...
*4283·96	1	0	4284·04	...	...	4284·04	2	Well defined	−0·08	...
4292·35	2–3	0	4292·09	4292·57	4292·36	4292·26	2–3	Well defined	+0·09	...
4300·37	2	0	4300·66	4300·10	4300·89	4300·64	1–2	Rather hazy	−0·27	Stellar line partly due to *p* Ti 4300·21.
4326·82	5	0	4326·68	4326·81	4327·03	4326·79	3	Well defined	+0·03	...
4344·19	8	0	4344·10	...	...	4344·10	2	Rather hazy	+0·09	Difficult. Inside fringe of Hγ.
4348·62	2	0	4348·30	4348·63	4348·84	4348·50	2	Well defined	+0·12	...
4365·50	1–2	0	4365·02	4365·87	4365·53	4365·29	1	Weak, well defined	+0·21	...
4478·86	2–3	0	4478·67	4478·55	4478·64	4478·64	1–2	Weak, well defined	+0·22	...
†4755·89	2	< 1	4755·79	...	...	4755·79	4	Well defined	+0·10	...
†4764·90	2	0	4764·62	...	...	4764·62	2	Hazy and broad	+0·28	...

three independent sets of measures, two on the best Kensington two-prism plate, and one on the best one-prism plate; the second are the means of measures on two Cambridge plates; and the third are from one set of measures on the best Kensington two-prism plate. The wave-lengths from the Cambridge plates are based on iron comparison spectra on the same plates, and a velocity correction deduced from lines of known origin at 4267·40 (carbon), H_γ, 4471·6 (helium), and 4481·4 (proto-magnesium). The wave-lengths from the Kensington plates are based on those of several well-authenticated lines (Ca 3933·83, Si 4128·20, *p*Fe 4233·33, C 4267·4, *p*Mg 4481·4, and H_β) in the stellar spectrum, Hartmann's formula being used for the reduction to wave-lengths of the remaining stellar lines. By taking into consideration the relative quality and dispersion of the plates measured, suitable weights have been given to the various wave-length reductions, and the weighted means are given in the seventh column. The eighth and ninth columns give respectively the intensity and description of the stellar lines. The tenth column gives the difference in wave-length between the proto-manganese lines and the stellar lines with which they are considered to be identical. The last column is reserved for remarks.

Column 10 shows that some of the differences between the estimated wave-lengths of the stellar and laboratory lines are rather large, the maximum being 0·45 tenth-metres. The average difference, however, for the twenty-three lines involved is 0·18 tenth-metre, and almost all the larger differences relate to lines which, from their nature and intensity in the stellar spectrum, are very difficult to bisect under the micrometer.

With the possible exception of line 4251·86, which falls in position on an apparently continuous but narrow band in the stellar spectrum (this band probably consists of several contiguous stellar lines merged together), there is no proto-manganese line which does not agree in position—within the limits of error in measurement—with an α Andromedæ line, and this long succession of close agreements leaves little or no doubt that the two sets are identical. Moreover, these are, in general, the stellar lines which make the α Andromedæ spectrum so peculiar and so divergent from the normal types.

Comparison of the Relative Intensities of the Proto-manganese and α Andromedæ Lines.

Although practically all the enhanced lines of manganese seem to be represented in the stellar spectrum, a comparison of their intensities shows they are far from being of the same relative intensity in the laboratory and celestial spectra. Thus the strongest enhanced line (4344·19, Int. 8) is comparatively weak (Int. 2) in the stellar spectrum, whereas lines 3943·93, 4137·16, 4206·56, and 4326·82, whose spark intensities are respectively 2, 3–4, 3–4, and 3, are represented in the star by lines of respective

intensity 4, 3–4, 3–4, and 3. This lack of agreement in relative intensity is probably due to the stellar conditions being so modified from the laboratory conditions as to alter to some extent the relative intensities of the enhanced lines. The same kind of thing occurs in connection with other stellar spectra and enhanced lines of other metals. Thus the two enhanced lines of iron, 4173·61, 4179·05, which are weak in the iron spark spectrum, are represented in α Cygni by two very strongly marked lines, whereas the well-known quartet of proto-iron lines at 4508·46, 4515·51, 4520·40, 4522·77, which are much stronger in the iron spark than the two lines previously mentioned, are in the star less conspicuous than the two lines 4173·61, 4179·05.

In this connection an interesting problem suggests itself: that is, to obtain, if possible, by variation of spark conditions, a laboratory photograph of the proto-manganese lines which shows the same relative intensities of the enhanced lines as they appear to have in the star. It may be, however, that the change in relative intensity is partly due to the presence of many other metallic vapours in the star, whereas in the laboratory investigations the pure metal is used whenever possible.

Reference to Proto-manganese Lines in other Stellar Spectra.

It is fairly certain that some of the enhanced lines of manganese which are well marked in α Andromedæ, occur as very weak lines in other stellar spectra. In a "Catalogue of 470 of the Brighter Stars," published by the Solar Physics Committee in 1902, detailed records are given of the wave-lengths of the lines in spectra of α Cygni, α Canis Majoris, β Orionis, ζ Tauri, γ Orionis, and ϵ Orionis. In the first two of these the following lines are ascribed to proto-manganese:—

α Cygni.			α Canis Majoris.		
4000·0	(1)	Possibly *p*Mn.	4000·0	(1)	Possibly *p*Mn.
4242·5	(3–4)	Possibly partly *p*Mn.	4206·5	(1)	Possibly *p*Mn.
4253·1	(2)	Possibly *p*Mn.	4242·6	(2)	Possibly partly *p*Mn.
4292·4	(1)	Possibly *p*Mn.	4253·1	(1–2)	Possibly *p*Mn.
4344·3	(2)	Probably partly *p*Mn.	4292·4	(2)	Possibly *p*Mn.
4365·4	(<1)	Possibly *p*Mn.	4344·3	(1–2)	Probably partly *p*Mn.

Although, then, the presence of the enhanced lines of manganese in stellar spectra has been already recorded, only a few of them appear to be represented, and then only by very weak and inconspicuous lines, in the spectra of such stars as α Cygni and α Canis Majoris. In α Andromedæ, however, they are practically all present, and form quite a marked feature of its spectrum. In α Cygni and α Canis Majoris the enhanced lines of iron, chromium, and titanium are strongly shown, the proto-manganese lines being in some cases lacking altogether, and in others comparatively weak and insignificant. In α Andromedæ the reverse is the case,

the enhanced lines of iron, chromium, and titanium occur only as weak lines, whereas the proto-manganese lines are, in general, comparatively strong and conspicuous.

Again, although the enhanced lines of such elements as magnesium, silicon, calcium, iron, titanium, and chromium are common to a great number of the brighter stars of types ranging from B8 to F5, the enhanced manganese lines seem to be quite special to α Andromedæ, of the brighter stars, in so far as their occurrence as well-marked lines is concerned. It may here be pointed out that μ Leporis, a fifth magnitude star, is recorded by Miss Maury as having a similar spectrum to that of α Andromedæ, so far as she could judge from a Harvard two-prism plate of that star. Apart from these two, there seems to be no stellar spectrum recorded of a similar type.

The proto-manganese lines apparently account for most of the peculiarities of the α Andromedæ spectrum, but there are a few fairly well-marked lines left which seem to be special to this type of spectrum and are still unidentified. Chief of these are lines at approximate wave-lengths 3984·1, 4033·1, 4044·9, 4085·4, 4171·8, 4239·1, 4255·9. Line 4033·1 is apparently in the same position as the second member of the strong manganese quartet 4030·88, 4033·22, 4034·64, 4035·80, but in the absence of the other three members the identification with manganese can hardly be accepted. Line 4171·8 is in close agreement with the α Cygni line 4172·0 ascribed to proto-titanium 4172·07, but its intensity in α Andromedæ is too great to be accounted for solely by proto-titanium, the other lines of that element being very feeble in α Andromedæ. In this particular stellar spectrum the two lines just mentioned (4033·1, 4171·8) are strongly suggestive of the two strong gallium lines 4033·19, 4172·25, and as Exner and Haschek record another spark line (weak) of gallium at 4085·45, which is in agreement with the fourth stellar line of those mentioned above, the evidence for this identity is somewhat strengthened. This can, however, only be considered as a provisional identification, to be further inquired into when more satisfactory stellar photographs and more knowledge of the enhanced lines of gallium are available.

In the examination of various plates of α Andromedæ in relation to this inquiry, it has been noted that some of the spectrum lines appear at times to alter their character, relative intensity, and possibly their position. There are not enough good plates in the series to settle these points definitely, but further research will be made on the α Andromedæ spectrum in general, and on these points in particular, when better spectra of the star are available.

Reference to Plate.

In the accompanying plate the α Andromedæ spectrum is compared with that of α Cygni in strips 1 to 4. Many of the lines are common to the two spectra, but the proto-manganese

lines, which are very weak or lacking in α Cygni, occur in α Andromedæ and are denoted by their wave-lengths placed above strips 1 and 3. Some of the proto-manganese lines are seen with difficulty in the print of the stellar spectrum, but are easily seen with small magnification in the original negative. The stellar prints in the plate are fivefold enlargements from a Kensington spectrum photographed with two 6-inch 45° Henry prisms and 6-inch objective.

In strips 5 to 8 the spark and arc spectra of manganese are compared, and the enhanced lines denoted by their wave-lengths placed above strips 5 and 7. In this case the prints prepared for reproduction were from direct contact with the original negatives, which were obtained with a 6-inch Rowland concave grating.

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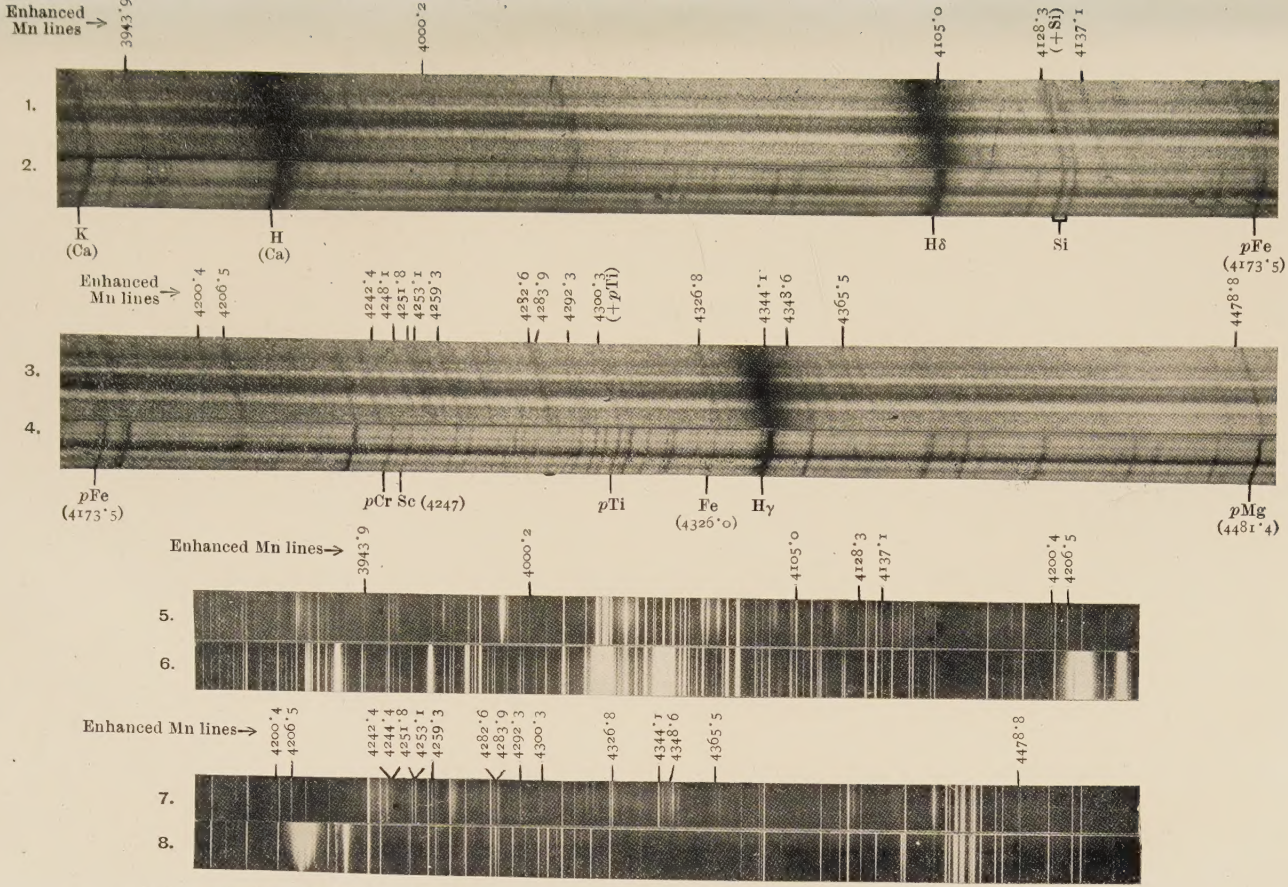

1. α Andromedæ (λλ 3930–4175).
2. α Cygni.
3. α Andromedæ (λλ 4170–4485).
4. α Cygni.
5. Mn Spark (λλ 3885–4230).
6. Mn Arc.
7. Mn Spark (λλ 4170–4520).
8. Mn Arc.